科技部補助專題研究計畫成果報告
期末報告

限量生產的行銷整合策略

計 畫 類 別 ： 個別型計畫
計 畫 編 號 ： MOST 103-2221-E-263-002-
執 行 期 間 ： 103 年 08 月 01 日至 104 年 07 月 31 日
執 行 單 位 ： 致理學校財團法人致理科技大學企業管理系

計 畫 主 持 人 ： 滕慧敏

計畫參與人員： 大專生-兼任助理人員：林妤

報 告 附 件 ： 出席國際會議研究心得報告及發表論文

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中 華 民 國 104 年 09 月 05 日

中文摘要： 所謂稀少性，「任何一種商品只要不易獲得，就會增加它的價值」。而限量策略則是廠商操弄產品的稀少性，企圖影響消費者的價值知覺，進而促使消費更渴望擁有的行銷手法，許多的名牌都於特定時點，推出限量商品。由於過往的研究，多從行銷面來探討限量發行對消費者購買意願的影響。本研究計畫擬針對產品的限量生產(發行)及限時折價，強調其稀少性以提高消費者的購買慾望，並建立模式以求取最佳的生產策略。

本研究乃假設產品單價隨產品總量增加而下降，而其銷售量又隨產品單價上升而下降，在此條件中產品總量(生產量)，使總利潤為最大研究方法主要採用存貨理論模式及相關機率與統計理論為基礎進行研究，建立問題的數學模式，並以利潤最大化（或成本最小化）為目標，運用最佳化理論找出最佳訂購/生產量、最佳缺貨量及最佳訂購/生產週期等。以數值範例說明所建立模型的應用情形，並對重要參數進行敏感度分析。本研究可提供供應商與零售商作為最佳政策決定之依據。

中文關鍵詞： 限量生產數量,報童,稀少性

英文摘要： An important strategy for dealing with scarcity and customer response is to produce limited quantity of certain products. Due to limited production quantity, consumers would feel the value or uniqueness of the products and have a stronger urgency to purchase them.
Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: both the unit selling price and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.

英文關鍵詞： Limited production quantity； Newsvendor； Scarcity

科技部專題研究計畫成果報告撰寫格式
科技部補助專題研究計畫成果報告

（□期中進度報告/■期末報告）

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本計畫除繳交成果報告外，另含下列出國報告，共乙份：

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■出席國際學術會議心得報告

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中　華　民　國　104　年 08　月 31 日

一、中文摘要

所謂稀少性，「任何一種商品只要不易獲得，就會增加它的價值」。而限量策略則是廠商操弄產品的稀少性，企圖影響消費者的價值知覺，進而促使消費更渴望擁有的行銷手法，許多的名牌都於特定時點，推出限量商品。由於過往的研究，多從行銷面來探討限量發行對消費者購買意願的影響。本研究計畫擬針對產品的限量生產(發行)及限時折價，強調其稀少性以提高消費者的購買慾望，並建立模式以求取最佳的生產策略。

本研究乃假設產品單價隨產品總量增加而下降，而其銷售量又隨產品單價上升而下降，在此條件中產品總量(生產量)，使總利潤為最大研究方法主要採用存貨理論模式及相關機率與統計理論為基礎進行研究，建立問題的數學模式，並以利潤最大化（或成本最小化）為目標，運用最佳化理論找出最佳訂購/生產量、最佳缺貨量及最佳訂購/生產週期等。以數值範例說明所建立模型的應用情形，並對重要參數進行敏感度分析。本研究可提供供應商與零售商作為最佳政策決定之依據。

二、英文摘要

An important strategy for dealing with scarcity and customer response is to produce limited quantity of certain products. Due to limited production quantity, consumers would feel the value or uniqueness of the products and have a stronger urgency to purchase them. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin.

In this study, we consider a newsvendor problem for products with limited production quantity: both the unit selling price and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.

三、中文關鍵字

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四、英文關鍵字

Limited production quantity; Newsvendor; Scarcity

附件二

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現（簡要敘述成果是否有嚴重損及公共利益之發現）或其他有關價值等，作一綜合評估。

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□ 未達成目標（請說明，以 100 字為限）

□ 實驗失敗

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本研究乃假設產品單價隨產品總量增加而下降，而其銷售量又隨產品單價上升而下降，在此條件中產品總量(生產量)，使總利潤為最大研究方法主要採用存貨理論模式及相關機率與統計理論為基礎進行研究，建立問題的數學模式，並以利潤最大化（或成本最小化）為目標，運用最佳化理論找出最佳訂購/生產量、最佳缺貨量及最佳訂購/生產週期等。以數值範例說明所建立模型的應用情形，並對重要參數進行敏感度分析。本研究可提供供應商與零售商作為最佳政策決定之依據。

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An optimization model for products with limited production quantity

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ABSTRACT

An important strategy for dealing with scarcity and customer response is to produce limited quantity of certain products. Due to limited production quantity, consumers would feel the value or uniqueness of the products and have a stronger urgency to purchase them. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: both the unit selling price and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical examples and sensitivity analysis are presented to illustrate the model.

1. Introduction

Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior [1]. Consumers often consider possessing scarce products to show their uniqueness, and it triggers them to desire these products. Consequently, some manufacturers may design their marketing strategies by producing a limited quantity of their products. This type of marketing strategy is very common among innovative products. For example, department stores announce limited products on their promotional flyers during their anniversary sales. Customers are required to book in advance or wait in line in order to buy the limited products. Besides, it is very common for some famous product brands to offer limited products during their seasonal sales to attract attention and sales. For another example, Motorola V3i cooperated with D&G to launch a limited version of luxury cellphone called Color Gold; they only produce 1000 pieces of such version worldwide, and its selling price is much higher than other unlimited production quantity version [2].

Commodity theory [1,2] deals with the psychological effects of scarcity. The theory claims that "any commodity will be valued if it is unavailable". According to the theory, scarcity enhances the value (or desirability), and it gives to its possessor a sense of pride in possessing the limited product [1]. The feeling of uniqueness may vary for different situations and persons; as such, it may be related to: (a) forces in a given situation that promote an extreme sense of high similarity, and (b) dispositional factors that influence the high need for uniqueness across a variety of situations [3]. Sirgy [4] addressed the importance of scarcity in marketing strategy. Salespersons should apply such strategy while merchandising products or services; it will increase the motivation of the targeted customers to approach the promotional information. There are two strategies for

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price raise through scarcity: (1) direct result from the quality and symbolic interest, and (2) indirect result on quality and symbolic interest through the price. As a result, raising the prices of scarce products can make a positive impact, but also may backfire if it is not launch properly [5]. Therefore, if we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.

Recently, many industries apply the strategy of limited production through single production schedule. In inventory management, this problem is known as the "newsboy problem" or the "newsvendor problem". The newsboy problem is a single period stochastic inventory problem [6,7] which deals with stocking issues in today's supply chains [8–10]. Weng [11] analyzed the coordinated quantity decisions between the manufacturer and the buyer in a newsvendor model. Dominey and Hill [12] explored the effectiveness of approximating a compound Poisson distribution in a newsboy model. Wang and Webster [13] used loss aversion to model manager's decision-making behavior in the single-period newsvendor problem. Shi et al. [14] extended the multi-product newsvendor problem by incorporating the retailer's pricing decision considering supplier quantity discount. From our literature search, no researches have been done on the newsvendor problem to consider the limited production quantity issues.

In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier. We present an algorithm to derive an optimal production quantity and selling price such that the expected profit is maximized.

2. Notations

The following notations are used in our analysis:

- $E\pi$ the expected profit for the supplier
- Q the production quantity for the supplier; decision variable
- Q^* the optimal production quantity for the supplier considering limited production quantity
- Q_w the production quantity for the supplier without considering limited production quantity
- Q_w^* the optimal production quantity for the supplier without considering limited production quantity
- p_1 the selling price per unit without considering limited production quantity; constant
- p_2 the upper bound of selling price per unit when the production quantity is limited; constant
- $p(Q)$ the selling price per unit with considering limited production quantity; which is a function of production quantity
- c_p the production price per unit; $c_p < p(Q)$
- s the salvage value per unit $s < c_p$
- r the shortage cost per unit; represents costs of lost goodwill
- x the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$

3. Modeling and assumptions

Throughout this study, single production of the limited product is assumed. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers. The unit production price of the product is c_p. For simplicity, the unit production cost is assumed to be constant. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q) = p_1$, the supplier will manufacture an optimal batch of Q_w^*. This is identical to the newsboy problem. The suppliers' expected profit function $E\pi$ is:

$$E\pi(Q_w) = \int_0^{Q_w} \{[p_1 - c_p]x - (c_p - s)(Q_w - x)\}f(x)dx + \int_{Q_w}^{\infty} \{[p_1 - c_p]Q_w - (x - Q_w)r\}f(x)dx. \tag{1}$$

Similar to Hadley and Whitin [15], the suppliers' optimal production batch is:

$$F(Q_w^*) = (p_1 - c_p + r)/(p_1 - s + r), \tag{2}$$

where $F(x)$ is the CDF of x. If the supplier manages the limited production batch, then the consumers' perceived value and purchase decisions are usually influenced by the law of scarcity [1]. The unit selling price $p(Q)$ of the limited quantity products is a decreasing function of Q. However, the customer demand will decrease due to a higher selling price. That is, the random demand of the products depends on production batch, Q, because the higher production batch will decrease the selling price, while the lower selling price will increase demand. That means the PDF, $f(x)$, of the random demand x is a function of Q. The suppliers' expected profit function $E\pi$ is given as follows:

$$E\pi(Q) = \int_0^{Q} \{[p(Q) - c_p]x - (c_p - s)(Q - x)\}f(x)dx + \int_{Q}^{\infty} \{[p(Q) - c_p]Q - (x - Q)r\}f(x)dx. \tag{3}$$

Our problem can be formulated as:

$$Max : E\pi(Q). \tag{4}$$

We illustrate the model by a case study.

4. An illustrative case study

In this section, a practical selling price and probability distribution are applied to explain the results of the previous section. Since the selling price is always influenced by the limited production quantity (Wu and Hsing [5]), the selling price per unit $p(Q)$ can therefore be assumed as:

$$p(Q) = \frac{p_2 - p_1}{\sqrt{Q}} + p_1, \quad p_2 > p_1 > 0, \quad Q \geqslant 1. \tag{5}$$

which means $p_1 < p(Q) < p_2$ and is a decreasing function of Q. For the supplier, the random demand is assumed to be uniformly distributed over the range 0 and $B(Q)$, where

$$B(Q) = \frac{bp_1}{p(Q)}, \tag{6}$$

is a function of Q with positive constant b (b is the upper bound of the selling quantity). This means that a higher selling price would decrease demand. Thus, the PDF of the supplier's demand is

$$f(x) = \frac{1}{B(Q)}. \tag{7}$$

Two cases are discussed as follows.

4.1. General case

In this case, $B(Q) = \frac{bp_1}{p(Q)}$, one has (calculated by mathematical software Maple 8)

$$E\pi'(Q) = \int_0^Q [p'(Q)x - c_p + s]\frac{1}{B(Q)}dx + \int_Q^{B(Q)} [p'(Q)Q + p(Q) - c_p + r]\frac{1}{B(Q)}dx \\ + B'(Q)f(B(Q))\{[p(Q) - c_p]Q - [B(Q) - Q]r\}. \tag{8}$$

$$E\pi''(Q) = p''(Q)\frac{Q^2}{2B(Q)} + [p''(Q)Q + 2p'(Q)]\frac{B(Q) - Q}{B(Q)} - [p(Q) - s + r]f(Q) \\ + \{[p(Q) - c_p]Q - [B(Q) - Q]r\}[B'(Q)]^2 f'(B(Q)) \\ + \left\{2B'(Q)[p'(Q)Q + p(Q) - c_p + r] + B''(Q)[p(Q)Q - c_pQ - rB(Q) + rQ] - r[B'(Q)]^2\right\}f(B(Q)). \tag{9}$$

From (9), it is hard to prove the concavity of $E\pi(Q)$. Two numerical examples are provided to illustrate the model.

Example 1. Given $p_2 = 250$, $p_1 = 130$, $c_p = 100$, $b = 1500$, $s = 50$, and $r = 5$, then (Please refer to Fig. 1)

$$E\pi(Q) = \left(3690180000Q^{\frac{1}{2}} - 154405544Q - 3539064Q^{\frac{3}{2}} + 9360Q^2 + 2873Q^{\frac{5}{2}}\right)\Big/\left[7800(12 + 13Q^{\frac{1}{2}})\right].$$

$$E\pi''(Q) = \Big(132846480000Q^{\frac{1}{2}} + 431751060000Q - 14243247360Q^{\frac{3}{2}} - 517035168Q^2 - 4836780Q^{\frac{5}{2}} \\ -3539367Q^3 - 971074Q^{\frac{7}{2}}\Big)\Big/\left[7800Q^2(12 + 13Q^{\frac{1}{2}})^3\right].$$

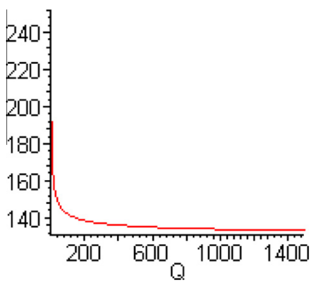

Fig. 1. The shape of $p(Q)$ in Example 1.

For the above equation of $E\pi''(Q)$, the first and third term, $(13284648000 - 1424324736Q)\,Q^{\frac{1}{2}}$, and the second and fourth term, $(43175106000 - 517035168Q)Q$ are negative since $Q > 84$. That means that $E\pi''(Q) < 0$, therefore $E\pi(Q)$ is concave. The concavity of $E\pi(Q)$ is also illustrated in Figs. 2–4. Fig. 2 presents the curve for $E\pi(Q)$; it reaches a maximum in the interval [500,1000]. Fig. 4 presents the curve for $E\pi''(Q)$ showing its negative value. It shows $E\pi(Q)$ is concave. Fig. 3 presents the curve for $E\pi'(Q)$; it shows the root of $E\pi'(Q) = 0$, and it is located in the interval [500,1000]. Setting $E\pi'(Q)$ equals to zero, $Q^* = 614$ is derived using Maple 8, the selling price per unit is $p(Q^*) = \$134.8$, and the optimal expected profit for the supplier is $E\pi(Q^*) = \$9137$. When limited production quantity is not considered, $Q_w^* = 618$ (using Eq. (2)), $E\pi(Q_w^*) = \$7059$ (using Eq. (1)), and the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1 = 29.4\%$.

Example 2. A company manufactures a new brand of basketball shoes with limited production lot size of 50,000 units. Given that the upper bound of selling price per unit is $p_2 = \$360$, the selling price per unit without considering limited production quantity is $p_1 = \$60$, the production price per unit is $c_p = \$40$, the upper bound of selling quantity is $b = \$50{,}000$, the salvage value per unit is $s = \$35$, and the shortage cost per unit is $r = \$2$.

From the above data, the expected profit is:

$$E\pi(Q) = \left(-40992500Q - 2196325Q^{\frac{2}{3}} + 570Q^2 + 27Q^{\frac{5}{2}} + 4850000000Q^{\frac{1}{2}}\right) \Big/ \left[100000\left(5 + Q^{\frac{1}{2}}\right)\right].$$

and

$$E\pi''(Q) = -\Big(4501631125Q^{\frac{3}{2}} + 30124875Q^2 + 35775Q^{\frac{5}{2}} + 4545Q^3 + 216Q^{\frac{7}{2}}$$
$$-72750000000Q - 121250000000Q^{\frac{1}{2}}\Big) \Big/ \left(400000Q^2\left(5 + Q^{\frac{1}{2}}\right)^3\right).$$

Both the first and seventh term, $(4501631125Q - 121250000000)Q^{1/2}$, and the second and sixth term, $(30124875Q - 72750000000)Q$ are positive since $Q > 2416$. This means that $E\pi''(Q) < 0$, resulting in the concavity of $E\pi(Q)$. By setting $E\pi'(Q)$ equal to zero, the optimal production quantity, $Q^* = 39{,}673$ is derived using Maple 8. The selling price per unit is $p(Q^*) = \$61.5$, and the optimal expected profit for the supplier is $E\pi(Q^*) = \$423{,}850$. When the limited production quantity is not considered, $Q_w^* = 40{,}741$ (using Eq. (2)), $E\pi(Q_w^*) = \$398{,}148$ (using Eq. (1)), the percentage profit increased is then $\frac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1 = 6.5\%$.

4.2. Special case

In this case, without considering a variable selling price, if $Q \to \infty$, then $p(Q) = p_1$, $B(Q) = b$, and $f(x) = \frac{1}{b}$. One has

$$E\pi'(Q) = \int_0^Q \left[-c_p + s\right]\frac{1}{b}dx + \int_Q^b \left[p_1 - c_p + r\right]\frac{1}{b}dx = \frac{-Q(p_1 - s + r)}{b} + (p_1 - c_p + r). \tag{10}$$

$$E\pi''(Q) = \frac{-(p_1 - s + r)}{b}. \tag{11}$$

Since, $p_1 - s > 0$, therefore, $E\pi''(Q) < 0$. This means that $E\pi(Q)$ is concave. The optimal solution Q^* of $E\pi(Q)$ is derived by setting $E\pi'(Q) = 0$. Thus,

$$Q^* = \frac{b(p_1 - c_p + r)}{p_1 - s + r}. \tag{12}$$

This result conforms to the solution of the traditional newsboy problem.

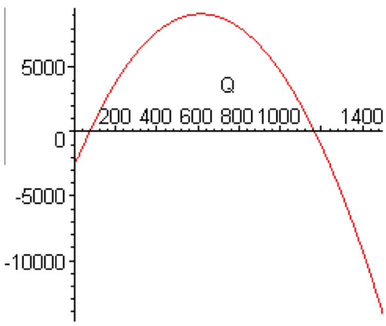

Fig. 2. The shape of $E\pi(Q)$ in Example 1.

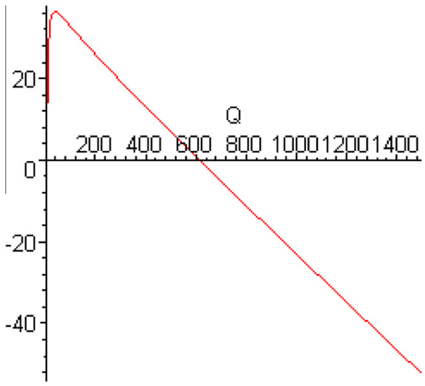

Fig. 3. The shape of $E\pi'(Q)$ in Example 1.

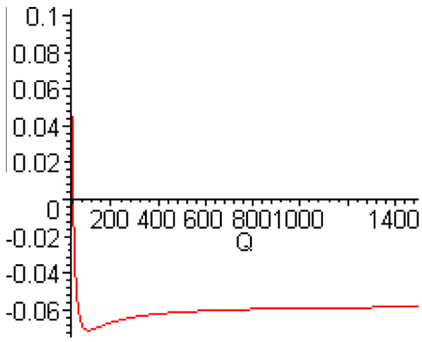

Fig. 4. The shape of $E\pi''(Q)$ in Example 1.

5. Sensitivity analysis

In order to analyze the effect of the limited production quantity, different parameters values in Example 1 are assumed. Tables 1–6 show the changes in Q^*, $p(Q^*)$, $E\pi(Q^*)$, and the % profit increase for variables p_2, p_1, c_o, b, s, and r, respectively. Table 1 shows the upper bound of the selling price (p_2) at $210, 220, \ldots, 290$, while the other variables remain unchanged. It is shown that as p_2 increases, the unit selling price, $p(Q^*)$, the expected profit, $E\pi(Q^*)$, and % profit increase all increases. However, in the real world, the price p_2 is usually regulated. Table 2 shows the upper bound of selling price (p_1) at $115, 120, \ldots, 155$, with the other variables remain unchanged. It is shown that as p_1 increases, the optimal production quantity, both Q^*, $p(Q^*)$, and $E\pi(Q^*)$ increases, but the % profit increase decreases.

Table 3 shows the unit production price (c_p) at $70, 75, \ldots, 110$, with the other variables remain unchanged. It is shown that as c_p increases, both the optimal production quantity, Q^*, $p(Q^*)$ and $E\pi(Q^*)$ increase, but the % profit increase decreases. Table 4 shows the unit production price (b) at $300, 600, \ldots, 2700$, with the other variables remain unchanged. It is shown that

Table 1
Sensitivity analysis for the upper bound of unit selling price p_2.

p_1 = 130, c_p = 100, b = 1500, s = 50, and r = 5				
p_2	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
210	616	133.2	8451	19.7
220	615	133.6	8623	22.2
230	615	134.0	8795	24.6
240	615	134.4	8966	27.0
250	614	134.8	9137	29.4
260	614	135.2	9308	31.9
270	614	135.7	9479	34.3
280	614	136.1	9649	36.7
290	613	136.5	9818	39.1

Note: % profit increase = $\left(\frac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1\right) * 100\%$.

Table 2
Sensitivity analysis for the unit selling price p_1.

$p_2 = 250$, $c_p = 100$, $b = 1500$, $s = 50$, $r = 5$				
p_1	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
115	443	121.4	2872	436.1
120	506	125.8	4770	90.8
125	563	130.3	6869	46.5
130	614	134.8	9137	29.4
135	661	139.5	11,552	20.5
140	703	144.1	14,092	15.2
145	741	148.9	16,741	11.6
150	776	153.6	19,485	9.1
155	809	158.3	22,312	7.3

Table 3
Sensitivity analysis for the unit production price c_p.

$p_2 = 250$, $p_1 = 130$, $b = 1500$, $s = 50$, $r = 5$				
c_p	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
70	1119	133.6	35,104	4.7
75	1034	133.7	29,723	6.1
80	949	133.9	24,765	7.9
85	865	134.1	20,229	10.5
90	781	134.3	16,114	14.1
95	698	134.5	12,417	19.8
100	614	134.8	9137	29.4
105	532	135.2	6272	49.6
110	450	135.7	3816	116.3

Table 4
Sensitivity analysis for the parameter b.

$p_2 = 250$, $p_1 = 130$, $c_p = 100$, $s = 50$, $r = 5$				
b	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
300	122	140.9	2327	64.8
600	245	137.7	4128	46.2
900	368	136.3	5839	37.9
1200	491	135.4	7503	32.9
1500	614	134.8	9137	29.4
1800	738	134.4	10,750	26.9
2100	861	134.1	12,347	24.9
2400	984	133.8	13,931	23.3
2700	1108	133.6	15,504	22.0

Table 5
Sensitivity analysis for the unit salvage value s.

$p_2 = 250$, $p_1 = 130$, $c_p = 100$, $b = 1500$, $r = 5$				
s	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
10	423	135.8	5531	53.6
20	459	135.6	6206	46.4
30	501	135.4	7005	40.1
40	552	135.1	7964	34.5
50	614	134.8	9137	29.4
60	693	134.6	10,609	24.8
70	796	134.2	12,509	20.5
80	934	133.9	15,065	16.3
90	1133	133.6	18,694	12.2

as b increases, both Q^* and $E\pi(Q^*)$ increase, but $p(Q^*)$ and the % profit increase decreases. Table 5 shows the unit production price (s) at $10, 20, \ldots, 90$, with the other variables remain unchanged. It is shown that as s increases, both Q^* and $E\pi(Q^*)$ increase, but $p(Q^*)$ and the % profit increase decreases. Table 6 shows the unit production price (r) at $1, 2, \ldots, 9$, with the other

Table 6
Sensitivity analysis for the unit shortage cost *r*.

$p_2 = 250$, $p_1 = 130$, $c_p = 100$, $b = 1500$, $s = 50$				
r	Q^*	$p(Q^*)$	$E\pi(Q^*)$	% profit increase
1	576	135	10,137	24.4
2	586	135	9879	25.6
3	596	134.9	9626	26.8
4	605	134.9	9379	28.1
5	614	134.8	9137	29.4
6	624	134.8	8901	30.9
7	633	134.8	8669	32.3
8	641	134.7	8443	33.9
9	650	134.7	8220	35.5

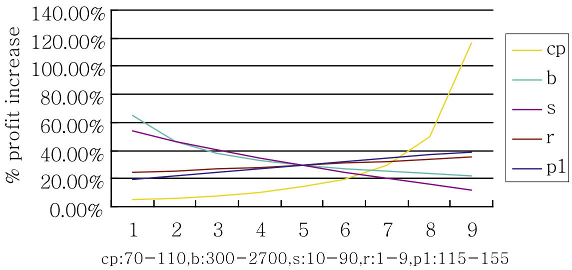

Fig. 5. The graphic presentation of sensitivity analysis in Example 1.

variables remain unchanged. It is shown that as r increases, both Q^* and the % profit increase, but $p(Q^*)$ and $E\pi(Q^*)$ decrease. The graphic presentation of the sensitivity analysis is illustrated in Fig. 5.

6. Conclusion

In this study, we derive a newsvendor problem model with limited production quantity. Limited production strategy allows the firm to manipulate the scarcity effect [2] of the products with limited production quantity. This enables the firm to increase the product selling prices due to exclusive distribution outlets. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.

Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model. The two numerical examples show that the percentage profit increase is fairly significant. Most of past researches focused on launch timing [16] and reciprocal effects [17]. The production and ordering quantities in this study directly influence the profit. For simplicity, the unit production cost is assumed to be constant. Future researches are suggested to consider variable unit production cost, selling price and varying distribution of customer demand.

Acknowledgments

The authors would like to thank Editor and anonymous referees for their valuable and constructive comments. This study was partially supported by Most 103-2221-E-263-002. They wish to express their deep appreciation to the National Science Council, ROC, for the financial support.

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附件五

科技部補助專題研究計畫出席國際學術會議心得報告

日期：103 年 12 月 01 日

計畫編號	MOST-103-2221-E-263 -002		
計畫名稱	限量生產的行銷整合策略		
出國人員姓名	滕慧敏	服務機構及職稱	致理技術學院企業管理系副教授
會議時間	103 年 11 月 06 日至 103 年 11 月 10 日	會議地點	日本廣島 廣島大學
會議名稱	(中文)第一屆東亞工業工程研討會 (英文) The 1st East Asia Workshop on Industrial Engineering(EAWIE)		
發表題目	(中文)限量產品的生產決策 (英文) Decisions for Products with Limited Production		

一、參加會議經過

詳如附件所示。

二、與會心得

本次研討會係由日本工業管理協會（JIMA）、 韓國工業工程學會（KIIE）及中國工業工程學會（CIIE）三個單位共同舉辦。感謝國科會對於此次國際會議的經費補助和支持，使我們有機會了解世界各地優秀學者的研究成果；尤其、此次主辦單位廣島大學是第一次舉辦國際研討會，該校負責單位的準備工作相當紮實，充分顯示出日本人的認真負責及細心。同時、在面對面的進行交流與觀摩當中，與會者所提出的最新成果和交流思想，對提升夠提升國內的研究水

準有相當大的助益。

另外、在會議中發表自己的研究成果，並和與會人士相互討論是非常難得的經驗，也提供了一些不同的思考模式，對於日後的研究方向有很大的幫助，且會議內容大部分都是尚未發表的研究成果，更啟發了我若干靈感，日後可豐富我的研究。

藉由這次研討會，增加英文論文發表及闡述之經驗，更可提昇未來在國際會議上的外語表達能力。

三、發表論文全文或摘要

詳如附件所列

四、建議

技職院校的經費及資源不足，研究工作推展不易。然而、不論大專或技職院校，私立學校任職的教師雖工作負荷很重；學生素質不高，研究助手難覓，但都有研究的意願及壓力，且各校都訂有提昇學術水準的目標。謝謝主管單位能給我機會，能在年過半百後，參與國際性的學術研討。懇請相關單位日後在分配經費時，能考慮多提供一些機會給私立院校的老師，感激不盡!

五、攜回資料名稱及內容

1、大會議程

2、論文摘要 USB

	Room WS1		Room WS2	
9:40-10:40	(JIMA session)			
	Procurement Logistics SU11 Chair: Koichi Nakade (Nagoya Institute of Technology)		Management Technology SU21 Chair: Hironobu Kawamura (University of Tsukuba)	
10:50-11:10	SU11-1 On Strengthening Procurement Management by "Dual Internal Control Method"	Xiaobing Liu (Dalian University of Technology), Haijun Liu (Dalian port technology Co., LTD.)	SU21-1 An Analysis of Purchasing and Browsing Histories on an EC Site Based on a New Latent Class Model	Masayuki Goto, Kenta Mikawa (Waseda University), Manabu Kobayashi (Shonan Institute of Technology), Shunsuke Horii, Tota Suko, Shigeichi Hirasawa (Waseda University)
11:10-11:30	SU11-2 Distribution Model of Disaster Relief Supplies by Considering Route Availability	Prudensy Opit, Koichi Nakade (Nagoya Institute of Technology)	SU21-2 A Multi-Period Model in Bankruptcy Prediction	Masahiro Koshika, Masanobu Matsumaru (Kanagawa University)
11:30-11:50	SU11-3 Developing an Order Quantity Allocation Model with the Consideration of Risks of Supply Quantity	Xiaobing Liu, Zhancheng Li, Liyuan Jiang, Li He (Dalian University of Technology)	SU21-3 Industrial Engineering Applications in Japanese-Style Inns	Yosuke Takada, Hironobu Kawamura (University of Tsukuba)
	Supply Chain Management SU12 Chair: Takashi Irohara (Sophia University)		Service/Starategy SU22 Chair: Kinya Tamaki (Aoyama Gakuin University)	
13:05-13:25	SU12-1 Analysis of Supply Chain Risk Management in the Japanese Automotive Industry	Munehiro Chino (Tokyo Metropolitan University), Yacob Khojasteh (Sophia University), Tetsuma Furuhata (Takachiho University), Yasutaka Kainuma (Tokyo Metropolitan University)	SU22-1 Investigation and Research for Global Product Strategy through Industry-University Project Group Activities	Kinya Tamaki (Aoyama Gakuin University), Y.W. Park(University of Tokyo), T. Abe, S. Goto(Aoyama Gakuin University)
13:25-13:45	SU12-2 The Effect of Customers' Active Responses to Product Unavailability on Supply Chain Coordination and Establishment of Brand Loyalty	Hisashi Kurata, Berdymyrat Ovezmyradov (University of Tsukuba)	SU22-2 Real Time Measurement and Analysis of Iku-men Activities for Childcare Service Innovation and Challenge	Tetsuo Yamada, Shigehiro Sakurada (University of Electro-Communications), Masato Takanokura (Kanawaga University), Seiko Taki (Chiba Institute of Technology), Tasuku Sato (University of Electro-Communications)
13:45-14:05	SU12-3 Coordination of Supply Chains with Multiple Members	Ilkyeong Moon (Seoul National University), Xuehao Feng (Zhejiang University), Youngsoo Park (Seoul National University), Younghoon Lee (Yonsei University)	SU22-3 A Framework of Integrated PLM System for International Production Strategy and Production Development	Masahiro Arakawa (Nagoya Institute of Technology)
	Production Management SU13 Chair: Chulung Lee (Korea University)		Supply Chain / Logistics SU23 Chair: Etsuko Kusukawa (Osaka Prefecture University)	
14:15-14:35	SU13-1 Optimum Arrangement and Effective Usage in Emergency Evacuation for e-Bikes	Shinya Mizuno (Shizuoka Institute of Science), Yasuyuki Muramatsu(Yamaha Motor Co., Ltd.), Naokazu Yamaki (Shizuoka University)	SU23-1 Analysis of Supply Coordination with Returns Handling and Discount Sales under E-Commerce Environment	Etsuko Kusukawa, Daiki Fujisono (Osaka Prefecture University)
14:35-14:55	SU13-2 Integrated Inventory and Capacity Decisions with Lateral Transshipments	Ki-sung Hong, In-Chan Choi (Korea University), Ilkyeong Moon (Seoul National University), Chulung Lee (Korea University)	SU23-2 Modeling Facility Location by Optimizing Time Performance for Humanitarian Relief Logistics	Wapee Manopiniwes, Keisuke Nagasawa, Takashi Irohara (Sophia University)
14:55-15:15	SU13-3 Research on Quality Management System for Diesel Manufacturers Based on the "Internet of Things"	Xiaobing Liu, Zhenyu Deng (Dalian University of Technology)	SU23-3 Scheduling Inter-Terminal Transshipment in Container Ports	Hak Bong Kim, Kap Hwan Kim (Pusan National University)
	Optimization SU14 Chair: Ping-Hui Hsu (De Lin Institute of Technology)		Product Development SU24 Chair: Jiahua Weng (Waseda university)	
15:25-16:40	SU14-1 A Study on Rules of Three Untrained Workers' Assignment Optimization under the Limited-Cycled Model with Multiple Periods	Peiya Song, Xianda Kong, Hisashi Yamamoto (Tokyo Metropolitan University), Jing Sun (Nagoya Institute of Technology), Masayuki Matsui (Kanagawa University)	SU24-1 A Proposal of Product Functional Structure Model for Engineer-to-Order Production	Shingo Akasaka, Jiahua Weng, Hisashi Ohnari (Waseda University)
15:45-16:05	SU14-2 Optimal Ordering Decision of Supply Chain by Increasing the Intermediary	Ping-Hui Hsu (De Lin Institute of Technology), Hui-Ming Teng(Chihlee Institute of Technology)	SU24-2 A Hybrid MCDM Model for New Product Development in the LiFePO4 battery Industry Using FDM, ISM and FANP	Wen Chen, Li Wang (Chung Hua University)
16:05-16:25	SU14-3 Decisions for Products with Limited Production	Hui-Ming Teng(Chihlee Institute of Technology), Ping-Hui Hsu(De Lin Institute of Technology), Hui-Ming Wee(Chung Yuan Christian University)	SU24-3 Disruption Management for Complex Flow Shop Scheduling with Considering Behavior	Hong-guang Bo, Yu-tao Pan, Xin Zhang, Xiao-yan Ma (Dalian University of Technology)
17:15-19:15	Workshop Dinner (Saijo HAKUWA Hotel)			

SU14-3

Decisions for Products with Limited Production

Hui-Ming Teng(Chihlee Institute of Technolog), Ping-Hui Hsu(De Lin Institute of Technology), Hui-Ming Wee(Chung Yuan Christian University)

Producing limited quantity of certain products is an important strategy for dealing with scarcity and customer response. Consumers often consider scarce products as possessing higher values, and it triggers them to desire these products. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical example is presented to demonstrate the model.

Full Paper

Decisions for Products with Limited Production

Hui-Ming Teng[†1], Ping-Hui Hsu[†2*] and Hui-Ming Wee[†3]

Abstract: Producing limited quantity of certain products is an important strategy for dealing with scarcity and customer response. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized.

Key words: Limited production quantity; Newsvendor; Scarcity

1. INTRODUCTION

Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior [1]. Consumers often consider scarce products as possessing higher values, and it triggers them to desire these products. Consequently, some manufacturers may design their marketing strategies by producing a limited quantity of their products. This type of marketing strategy is very common among innovative products. For example, department stores announce limited products on their promotional flyers during their anniversary sales. Customers are required to book in advance or wait in line in order to buy the limited products.

Commodity theory [2, 7] claims that "any commodity will be valued if it is unavailable". According to the theory, scarcity enhances the value (or desirability), and it gives to its possessor a sense of pride in possessing the limited product [1]. Sirgy [3] addressed the importance of scarcity in marketing strategy. Salespersons should apply such strategy while merchandising products or services; it will increase the motivation of the targeted customers to approach the promotional information. There are two strategies for price raise through scarcity: (1) direct result from the quality and symbolic interest, and (2) indirect result on quality and symbolic interest through the price. As a result, raising the prices of scarce products can make a positive impact, but also may backfire if it is not launch properly [4]. Therefore, if we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.

Recently, many industries apply the strategy of limited production through single production schedule. From our literature search, no researches have been done on the newsvendor problem to consider the limited production quantity issues. In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier. We present an algorithm to derive an optimal production quantity and selling price such that the expected profit is maximized.

2. NOTATIONS

The following notations are used in our analysis:

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the optimal production quantity for the supplier considering limited production quantity
Q_w	the production quantity for the supplier without considering limited production quantity
Q_w^*	the optimal production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity ; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant
$p(Q)$	the selling price per unit with considering limited production quantity ; which is a function of production quantity
$C_p(Q)$	the production cost per unit; which is a function of production quantity, $c_p(Q) < p(Q)$.
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
c_a	the marketing cost
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$

[†1] Chihlee Institute of Technology
[†2] De Lin Institute of Technology
[†3] Chung Yuan Christian University

3. MODELING AND ASSUMPTIONS

Throughout this study, single production of the limited product is assumed. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers. The unit production price of the product is c_p. For simplicity, the unit production cost is assumed to be constant. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q)= p_1$, the supplier will manufacture an optimal batch of Q^*_w . This is identical to the newsboy problem. The suppliers' expected profit function $E\pi$ is:

$$E\pi(Q_w) = \int_0^{Q_w} \left\{ \left[p_1 - c_p \right] x - (c_p - s)(Q_w - x) \right\} f(x) dx + \int_{Q_w}^{\infty} \left\{ \left[p_1 - c_p \right] Q_w - (x - Q_w) r \right\} f(x) dx. \quad (1)$$

Similar to Hadley and Whitin (1963), the suppliers' optimal production batch is:

$$F(Q^*_w) = (p_1 - c_p + r)/(p_1 - s + r), \quad (2)$$

where $F(x)$ is the CDF of x. If the supplier manages the limited production batch, then the consumers' perceived value and purchase decisions are usually influenced by the law of scarcity [17]. The unit selling price $p(Q)$ of the limited quantity products is a decreasing function of Q. However, the customer demand will decrease due to a higher selling price. That is, the random demand of the products depends on production batch, Q, because the higher production batch will decrease the selling price, while the lower selling price will increase demand. That means the PDF, $f(x)$, of the random demand x is a function of Q. The suppliers' expected profit function $E\pi$ is given as follows:

$$E\pi(Q) = \int_0^{Q} \left\{ \left[p(Q) - C_p(Q) \right] x - \left[C_p(Q) - s \right] (Q - x) \right\} f(x) dx + \int_{Q}^{\infty} \left\{ \left[p(Q) - C_p(Q) \right] Q - (x - Q) r \right\} f(x) dx - c_a. \quad (3)$$

Our problem can be formulated as:

$$Max : E\pi(Q). \quad (4)$$

We illustrate the model by a case study.

4. AN ILLUSTRATIVE CASE STUDY

In this section, a practical selling price and probability distribution are applied to explain the results of the previous section. Since the selling price is always influenced by the limited production quantity (Wu & Hsing, [5]), the selling price per unit $p(Q)$ can therefore be assumed as:

$$p(Q) = \frac{p_2 - p_1}{\sqrt{Q}} + p_1 , \; p_2 > p_1 > 0 , \; Q \geq 1, \quad (5)$$

which means $p_1 < p(Q) < p_2$ and is a decreasing function of Q (Please refer to Figure 1a). For the supplier, the random demand is assumed to be uniformly distributed over the range 0 and $B(Q)$, where

$$B(Q) = \frac{b p_1}{p(Q)} , \quad (6)$$

is a function of Q with positive constant b (b is the upper bound of the selling quantity). This means that a higher selling price would decrease demand. Thus, the PDF of the supplier's demand is

$$f(x) = \frac{1}{B(Q)} . \quad (7)$$

In the same time, the production cost is always influenced by the limited production quantity, the production cost per unit

$$C_p(Q) = \frac{c_{p2} - c_{p1}}{Q} + c_{p1} , \; c_{p2} > c_{p1} > 0 , \; Q \geq 1, \quad (8)$$

which means $c_{p1} < C_p(Q) < c_{p2}$ and is a decreasing function of Q (Please refer to Figure 1b). Obviously, $C_p(Q) < p(Q)$.

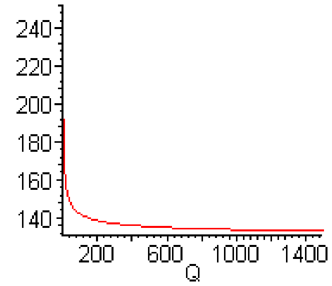

Fig. 1a. The shape of $p(Q)$, 0<Q<1500 in example 1

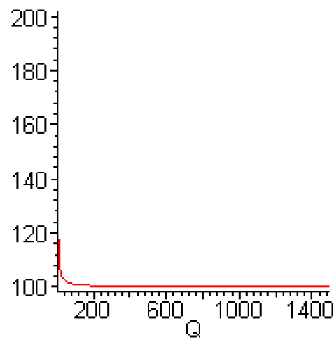

Fig. 1b. The shape of $Cp(Q)$, 0<Q<1500 in example 1

From, $B(Q) = \frac{bp_1}{p(Q)}$, one has (Calculated by mathematical software Maple 8)

$$E\pi'(Q) = \int_0^Q [p'(Q)x - c_p + s]\frac{1}{B(Q)}dx + \int_Q^{B(Q)} [p'(Q)Q + p(Q) - c_p + r]\frac{1}{B(Q)}dx + B'(Q)f(B(Q))\{[p(Q) - c_p]Q - [B(Q) - Q]r\}. \quad (9)$$

$$E\pi''(Q) = p''(Q)\frac{Q^2}{2B(Q)} + \left[p''(Q)Q + 2p'(Q)\right]\frac{B(Q)-Q}{B(Q)} - [p(Q) - s + r]f(Q) + \{[p(Q) - c_p]Q - [B(Q) - Q]r\}\left[B'(Q)\right]^2 f'(B(Q)) + \left\{2B'(Q)\left[p'(Q)Q + p(Q) - c_p + r\right] + B''(Q)\left[p(Q)Q - c_pQ - rB(Q) + rQ\right] - r\left[B'(Q)\right]^2\right\} f(B(Q)). \quad (10)$$

From (9), it is hard to prove the concavity of $E\pi(Q)$. A numerical example is provided to illustrate the model.

Example 1. Given p_2=250, p_1=130, c_p=100, c_{p1}=100, c_{p2}=200, b=1500, s=50, r=5 and c_a =1000, then

$$E\pi(Q) = -(480558000Q^{\frac{1}{2}} - 15440544Q - 3539064Q^{\frac{3}{2}} + 9360Q^2 + 2873Q^{\frac{5}{2}} + 102960000)/[7800(12 + 13Q^{\frac{1}{2}})]$$

$$E\pi'(Q) = -(1107054000Q^{\frac{1}{2}} - 926432264Q - 82033344Q^{\frac{3}{2}} - 22891596Q^2 + 134355Q^{\frac{5}{2}} + 37349Q^3)/[3900Q(12 + 13Q^{\frac{1}{2}})^2]$$

$$E\pi''(Q) = (13284648000Q^{\frac{1}{2}} + 43175106000Q - 1424324736Q^{\frac{3}{2}} - 517035168Q^2 - 4836780Q^{\frac{5}{2}} - 3539367Q^3 - 971074Q^{\frac{7}{2}})/[7800Q^2(12 + 13Q^{\frac{1}{2}})^3] \cdot$$

For the above equation of $E\pi''(Q)$, the first and third term, (13284648000-1424324736Q) $Q^{\frac{1}{2}}$, and the second and fourth term, (43175106000-517035168Q)Q are negative since Q>84. That means that $E\pi''(Q)$ <0, therefore $E\pi(Q)$ is concave. The concavity of $E\pi(Q)$ is also illustrated in Figures 2, 3, and 4. Figure 2 presents the curve for $E\pi(Q)$; it reaches a maximum in the interval [500, 800]. Figure 4 presents the curve for $E\pi''(Q)$ showing its negative value. It shows $E\pi(Q)$ is concave on [50,1500]. Figure 3 presents the curve for $E\pi'(Q)$; it shows the root of $E\pi'(Q)$=0, and it is located in the interval [500, 1000]. Setting $E\pi'(Q)$ equals to zero, Q^*=614 is derived using Maple 8, the selling price per unit is $p(Q^*)$=\$134.8, and the optimal expected profit for the supplier is $E\pi(Q^*)$ =\$8037. When limited production quantity is not considered, Q_w^* =618 (using Eq.(2)), $E\pi(Q_w^*)$ =\$7059 (using Eq.(1)), and the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1$ =13.9%.

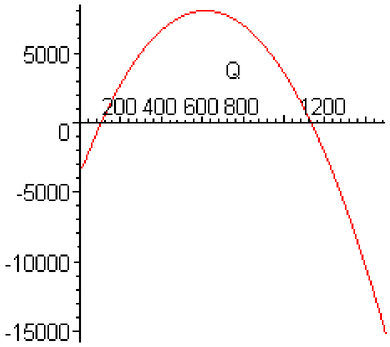

Fig. 2. The shape of $E\pi(Q)$, 0<Q<1500 in example 1

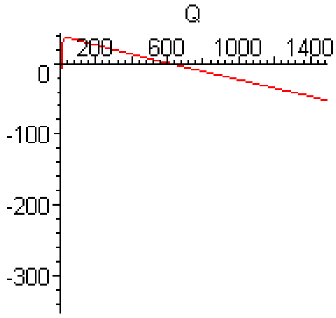

Fig. 3. The shape of $E\pi'(Q)$, 0<Q<1500 in example 1

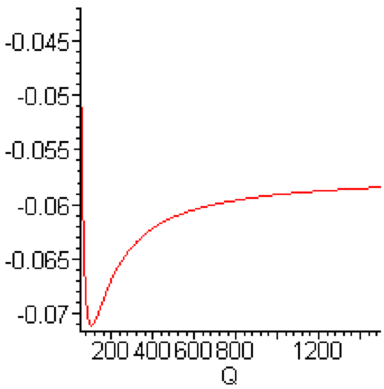

Fig. 4. The shape of $E\pi''(Q)$, 50<Q<1500 in example 1

5. CONCLUSION

Limited production strategy allows the firm to manipulate the scarcity effect [2] of the products with limited production quantity. This enables the firm to increase the product selling prices due to exclusive distribution outlets. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.

Illustrative case study and numerical example are presented to demonstrate the proposed model. The numerical example show that the percentage profit increase is fairly significant. Most of past researches focused on launch timing [5] and reciprocal effects [6]. The production and ordering quantities in this study directly influence the profit. Future researches are suggested to consider varying distribution of customer demand.

ACKNOWLEDGMENTS

This work was supported by MOST 103-2221-E-263-002, they wish to express their deep appreciation to the National Science Council, ROC, for the financial support.

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Decisions for Products with Limited Production

Author：Hui-Ming Teng
Ping-Hui Hsu
Hui-Ming Wee
Presenter： Hui-Ming Teng
Date ：2014/10/07

Agenda

1. ABSTRACT
2. INTRODUCTION
3. NOTATIONS
4. MODELING AND ASSUMPTIONS
5. AN ILLUSTRATIVE CASE STUDY
6. CONCLUSION

1、ABSTRACT

1. Producing limited quantity of certain products is an important strategy for **dealing with scarcity and customer response**.
2. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin.
3. In this study**, we consider a newsvendor problem for products with limited production quantity**: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity.
4. An algorithm is developed to obtain a production policy such that the expected profit is maximized.

2. INTRODUCTION(1)

- Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior(Lynn, M. 1991)
- Commodity theory claims that "any commodity will be valued if it is unavailable".
- There are two strategies for price raise through scarcity:

  (1) direct result from the quality and symbolic interest,

  (2) indirect result on quality and symbolic interest through the price.

2. INTRODUCTION(2)

- If we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.
- In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier.

3.NOTATIONS(I)

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the optimal production quantity for the supplier considering limited production quantity
Q_w	the production quantity for the supplier without considering limited production quantity
Q_w^*	the optimal production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity ; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant

3.NOTATIONS(II)

$p(Q)$	the selling price per unit with considering limited production quantity ; which is a function of production quantity
$c_p(Q)$	the production cost per unit; which is a function of production quantity, $c_p(Q) < p(Q)$.
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
c_a	the marketing cost
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$.

4、MODELING AND ASSUMPTIONS

1. Single production of the limited product is assumed.
2. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers.
3. The unit production price of the product is c_p. the unit production cost is assumed to be constant.
4. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s.
5. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q)=P_l$, the supplier will manufacture an optimal batch of Q_w^*.

Demand < Production quantity

$$E\pi(Q)=\int_0^Q\left\{\left[p(Q)-C_p(Q)\right]x-\left[C_p(Q)-s\right](Q-x)\right\}f(x)dx$$

$$+\int_Q^\infty\left\{\left[p(Q)-C_p(Q)\right]Q-(x-Q)r\right\}f(x)dx-c_a. \qquad (3)$$

Demand > Production quantity

Our problem can be formulated as:

$$Max: E\pi(Q). \qquad (4)$$

Since the selling price is always influenced by the limited production quantity (Wu & Hsing, [5]), the selling price per unit $p(Q)$ car

$$p(Q) = \frac{p_2}{\ldots}$$

which means p_1

Q |For the supp

uniformly distrib

$$B(Q) = \frac{\ldots}{p(Q)}$$

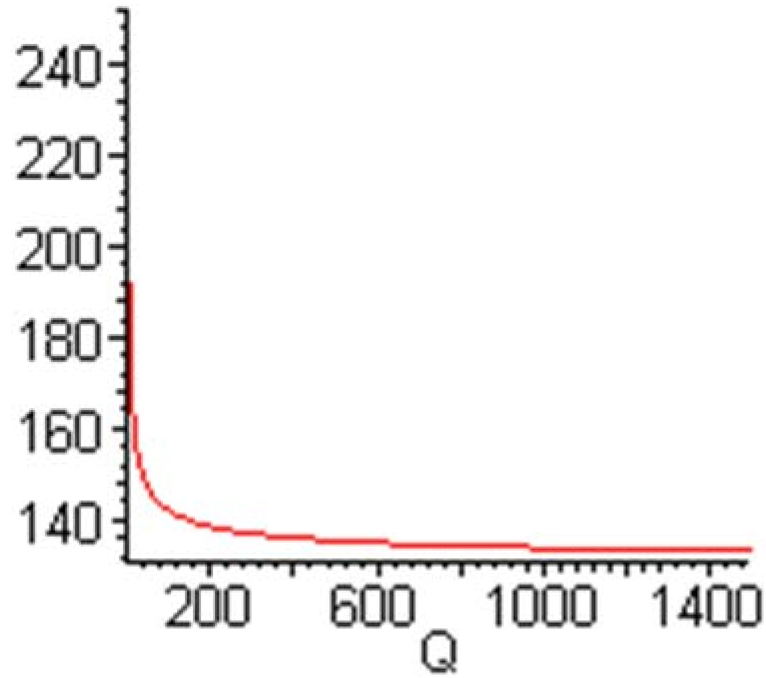

Fig. 1a. The shape of $p(Q)$, 0<Q<1500
in example 1

This means that a higher selling price would decrease de-
mand

(7)

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$C_p(Q)$

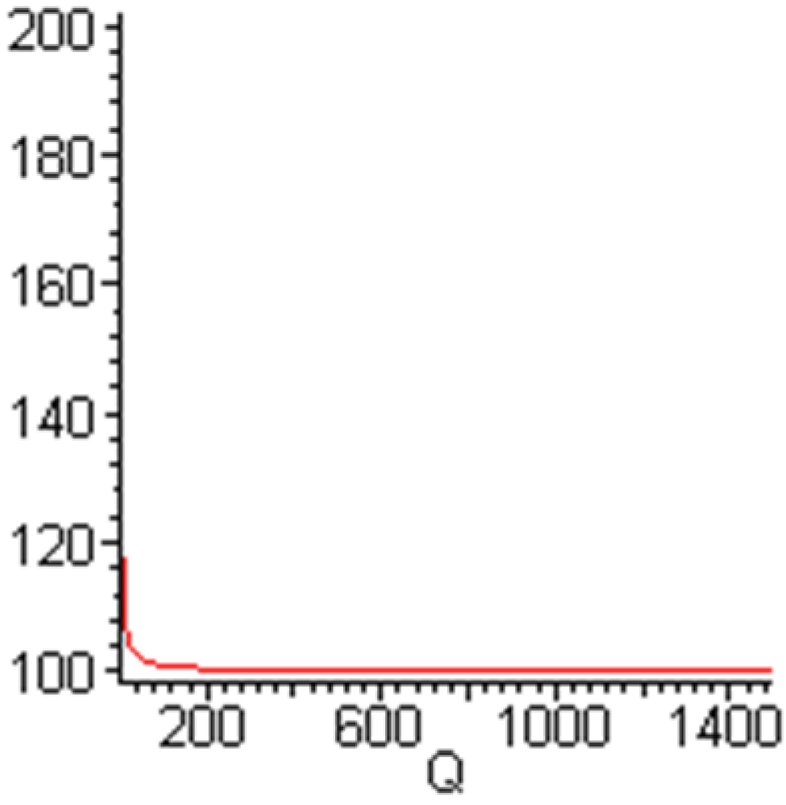

Fig. 1b. The shape of $Cp(Q)$, 0<Q<1500 in example 1

$$E\pi'(Q) = \int_0^Q [p'(Q)x - c_p + s]\frac{1}{B(Q)}dx + \int_Q^{B(Q)} [p'(Q)Q + p(Q) - c_p + r]\frac{1}{B(Q)}dx + B'(Q)f(B(Q))\left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}. \qquad (9)$$

$$E\pi''(Q) = p''(Q)\frac{Q^2}{2B(Q)} + \left[p''(Q)Q + 2p'(Q)\right]\frac{B(Q) - Q}{B(Q)} - \left[p(Q) - s + r\right]f(Q) + \left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}\left[B'(Q)\right]^2 f'(B(Q))$$

$$+\left\{2B'(Q)\left[p'(Q)Q + p(Q) - c_p + r\right] + B''(Q)\left[p(Q)Q - c_pQ - rB(Q) + rQ\right] - r\left[B'(Q)\right]^2\right\}f(B(Q)). \qquad (10)$$

From (9), it is hard to prove the concavity of $E\pi(Q)$. A numerical example is provided to illustrate the model.

Example 1. Given p_2=250, p_1=130, c_p=100, c_{p1}=100, c_{p2}=200, b=1500, s=50, r=5 and

c_a =1000, then

$$E\pi(Q) = -(480558000Q^{\frac{1}{2}} - 15440544Q - 3539064Q^{\frac{3}{2}} + 9360Q^{2} + 2873Q^{\frac{5}{2}} + 102960000)/[7800(12+13Q^{\frac{1}{2}})]$$

$$E\pi'(Q) = -(1107054000Q^{\frac{1}{2}} - 92643264Q - 82033344Q^{\frac{3}{2}} - 22891596Q^{2} + 134355Q^{\frac{5}{2}} + 37349Q^{3})/[3900Q(12+13Q^{\frac{1}{2}})^{2}]$$

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For $E\pi''(Q)$, the first and third term,

$(13284648000\text{-}1424324736Q)Q^{\frac{1}{2}}$,

and the second and fourth term,

$(43175106000\text{-}517035168Q)Q$

are negative since $Q>84$.

That means that $E\pi''(Q)<0$, therefore $E\pi(Q)$ is concave.

The concavity of $E\pi(Q)$ is illustrated in Figures 2, 3, and 4.

Figure 2 presents the curve for $E\pi(Q)$

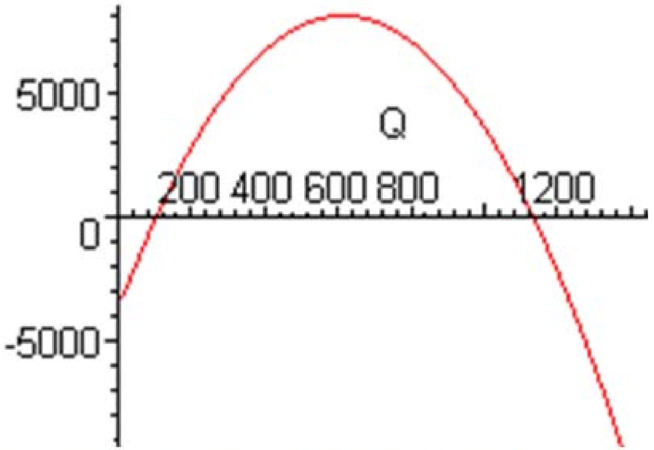

Fig. 2. The shape of $E\pi(Q)$, 0<Q<1500
in example 1

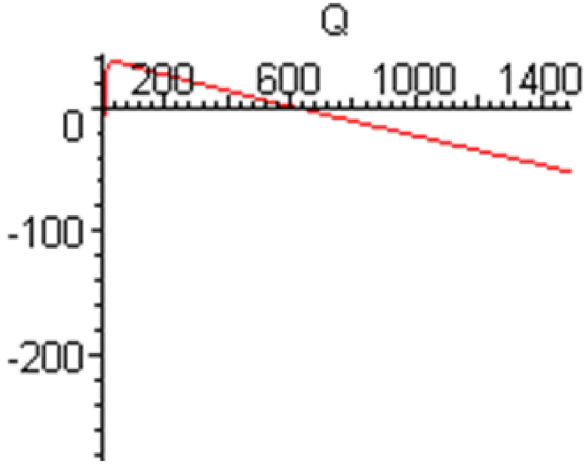

Fig. 3. The shape of $E\pi^{'}(Q)$, 0<Q<1500
in example 1

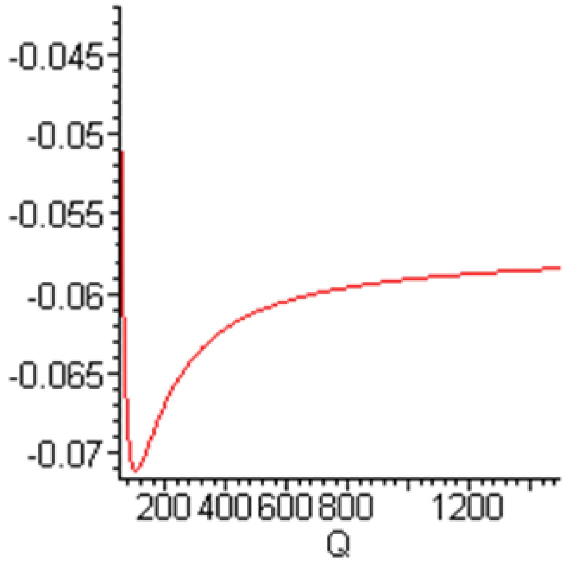

Fig. 4. The shape of $E\pi''(Q)$, 50<Q<1500
in example 1

Setting $E\pi'(Q)$ equals to zero, Q^*=614 is derived using Maple 8,

the selling price per unit is $p(Q^*)$=$134.8,

the optimal expected profit for the supplier is $E\pi(Q^*)$ =$8037.

When limited production quantity is not considered, Q_w^* =618

(using Eq.(2)),

$E\pi(Q_w^*)$ =$7059 (using Eq.(1)),

and the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)}-1$ =13.9%.

6.CONCLUSION(1)

1. Limited production strategy allows the firm to manipulate the scarcity effect [2] of the products with limited production quantity.
2. This enables the firm to increase the product selling prices due to exclusive distribution outlets.
3. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.
4. Illustrative case study and numerical example are presented to demonstrate the proposed model.

6.CONCLUSION(2)

5. The numerical example show that the percentage profit increase is fairly significant.
6. Most of past researches focused on launch timing [5] and reciprocal effects [6].
7. The production and ordering quantities in this study directly influence the profit. Future researches are suggested to consider varying distribution of customer demand.

Q &A


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4 · MODELING AND ASSUMPTIONS
1.Single production of the limited product is assumed.
2.The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers.
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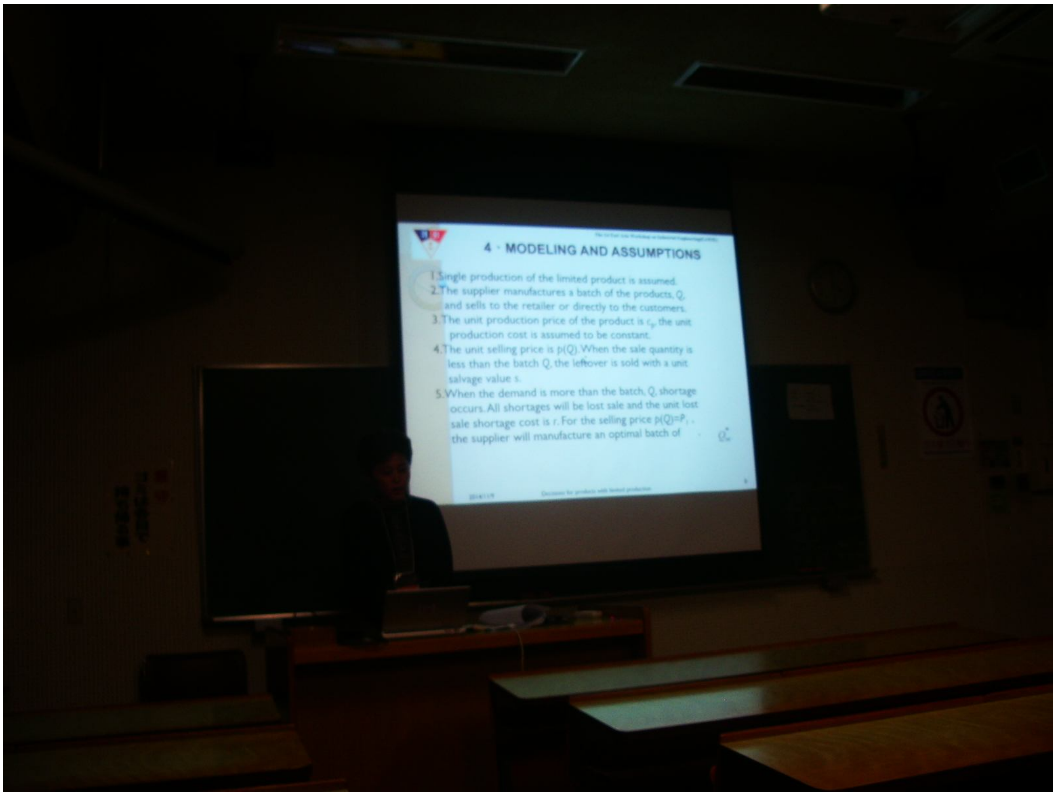
5.When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price p(Q)=P, , the supplier will manufacture an optimal batch of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附件五

科技部補助專題研究計畫出席國際學術會議心得報告

日期：103 年 12 月 01 日

計畫編號	MOST-103-2221-E-263 -002		
計畫名稱	限量生產的行銷整合策略		
出國人員姓名	滕慧敏	服務機構及職稱	致理技術學院企業管理系副教授
會議時間	103 年 11 月 06 日至 103 年 11 月 10 日	會議地點	日本廣島 廣島大學
會議名稱	(中文)第一屆東亞工業工程研討會 (英文) The 1st East Asia Workshop on Industrial Engineering(EAWIE)		
發表題目	(中文)限量產品的生產決策 (英文) Decisions for Products with Limited Production		

一、參加會議經過

詳如附件所示。

二、與會心得

本次研討會係由日本工業管理協會（JIMA）、 韓國工業工程學會（KIIE）及中國工業工程學會（CIIE）三個單位共同舉辦。感謝國科會對於此次國際會議的經費補助和支持，使我們有機會了解世界各地優秀學者的研究成果；尤其、此次主辦單位廣島大學是第一次舉辦國際研討會，該校負責單位的準備工作相當紮實，充分顯示出日本人的認真負責及細心。同時、在面對面的進行交流與觀摩當中，與會者所提出的最新成果和交流思想，對提升夠提升國內的研究水

準有相當大的助益。

另外、在會議中發表自己的研究成果，並和與會人士相互討論是非常難得的經驗，也提供了一些不同的思考模式，對於日後的研究方向有很大的幫助，且會議內容大部分都是尚未發表的研究成果，更啟發了我若干靈感，日後可豐富我的研究。

藉由這次研討會，增加英文論文發表及闡述之經驗，更可提昇未來在國際會議上的外語表達能力。

三、發表論文全文或摘要

詳如附件所列

四、建議

技職院校的經費及資源不足，研究工作推展不易。然而、不論大專或技職院校，私立學校任職的教師雖工作負荷很重；學生素質不高，研究助手難覓，但都有研究的意願及壓力，且各校都訂有提昇學術水準的目標。謝謝主管單位能給我機會，能在年過半百後，參與國際性的學術研討。懇請相關單位日後在分配經費時，能考慮多提供一些機會給私立院校的老師，感激不盡!

五、攜回資料名稱及內容

1、大會議程

2、論文摘要 USB

	Room WS1		Room WS2	
9:40-10:40	(JIMA session)			
	Procurement Logistics SU11		Management Technology SU21	
	Chair: Koichi Nakade (Nagoya Institute of Technology)		Chair: Hironobu Kawamura (University of Tsukuba)	
10:50-11:10	SU11-1 On Strengthening Procurement Management by "Dual Internal Control Method"	Xiaobing Liu (Dalian University of Technology), Haijun Liu (Dalian port technology Co., LTD.)	SU21-1 An Analysis of Purchasing and Browsing Histories on an EC Site Based on a New Latent Class Model	Masayuki Goto, Kenta Mikawa (Waseda University), Manabu Kobayashi (Shonan Institute of Technology), Shunsuke Horii, Tota Suko, Shigeichi Hirasawa (Waseda University)
11:10-11:30	SU11-2 Distribution Model of Disaster Relief Supplies by Considering Route Availability	Prudensy Opit, Koichi Nakade (Nagoya Institute of Technology)	SU21-2 A Multi-Period Model in Bankruptcy Prediction	Masahiro Koshika, Masanobu Matsumaru (Kanagawa University)
11:30-11:50	SU11-3 Developing an Order Quantity Allocation Model with the Consideration of Risks of Supply Quantity	Xiaobing Liu, Zhancheng Li, Liyuan Jiang, Li He (Dalian University of Technology)	SU21-3 Industrial Engineering Applications in Japanese-Style Inns	Yosuke Takada, Hironobu Kawamura (University of Tsukuba)
	Supply Chain Management SU12		Service/Starategy SU22	
	Chair: Takashi Irohara (Sophia University)		Chair: Kinya Tamaki (Aoyama Gakuin University)	
13:05-13:25	SU12-1 Analysis of Supply Chain Risk Management in the Japanese Automotive Industry	Munehiro Chino (Tokyo Metropolitan University), Yacob Khojasteh (Sophia University), Tetsuma Furuhata (Takachiho University), Yasutaka Kainuma (Tokyo Metropolitan University)	SU22-1 Investigation and Research for Global Product Strategy through Industry-University Project Group Activities	Kinya Tamaki (Aoyama Gakuin University), Y.W. Park(University of Tokyo), T. Abe, S. Goto(Aoyama Gakuin University)
13:25-13:45	SU12-2 The Effect of Customers' Active Responses to Product Unavailability on Supply Chain Coordination and Establishment of Brand Loyalty	Hisashi Kurata, Berdymyrat Ovezmyradov (University of Tsukuba)	SU22-2 Real Time Measurement and Analysis of Iku-men Activities for Childcare Service Innovation and Challenge	Tetsuo Yamada, Shigehiro Sakurada (University of Electro-Communications), Masato Takanokura (Kanawaga University), Seiko Taki (Chiba Institute of Technology), Tasuku Sato (University of Electro-Communications)
13:45-14:05	SU12-3 Coordination of Supply Chains with Multiple Members	Ilkyeong Moon (Seoul National University), Xuehao Feng (Zhejiang University), Youngsoo Park (Seoul National University), Younghoon Lee (Yonsei University)	SU22-3 A Framework of Integrated PLM System for International Production Strategy and Production Development	Masahiro Arakawa (Nagoya Institute of Technology)
	Production Management SU13		Supply Chain / Logistics SU23	
	Chair: Chulung Lee (Korea University)		Chair: Etsuko Kusukawa (Osaka Prefecture University)	
14:15-14:35	SU13-1 Optimum Arrangement and Effective Usage in Emergency Evacuation for e-Bikes	Shinya Mizuno (Shizuoka Institute of Science), Yasuyuki Muramatsu(Yamaha Motor Co., Ltd.), Naokazu Yamaki (Shizuoka University)	SU23-1 Analysis of Supply Coordination with Returns Handling and Discount Sales under E-Commerce Environment	Etsuko Kusukawa, Daiki Fujisono (Osaka Prefecture University)
14:35-14:55	SU13-2 Integrated Inventory and Capacity Decisions with Lateral Transshipments	Ki-sung Hong, In-Chan Choi (Korea University), Ilkyeong Moon (Seoul National University), Chulung Lee (Korea University)	SU23-2 Modeling Facility Location by Optimizing Time Performance for Humanitarian Relief Logistics	Wapee Manopiniwes, Keisuke Nagasawa, Takashi Irohara (Sophia University)
14:55-15:15	SU13-3 Research on Quality Management System for Diesel Manufacturers Based on the "Internet of Things"	Xiaobing Liu, Zhenyu Deng (Dalian University of Technology)	SU23-3 Scheduling Inter-Terminal Transshipment in Container Ports	Hak Bong Kim, Kap Hwan Kim (Pusan National University)
	Optimization SU14		Product Development SU24	
	Chair: Ping-Hui Hsu (De Lin Institute of Technology)		Chair: Jiahua Weng (Waseda university)	
15:25-16:40	SU14-1 A Study on Rules of Three Untrained Workers' Assignment Optimization under the Limited-Cycled Model with Multiple Periods	Peiya Song, Xianda Kong, Hisashi Yamamoto (Tokyo Metropolitan University), Jing Sun (Nagoya Institute of Technology), Masayuki Matsui (Kanagawa University)	SU24-1 A Proposal of Product Functional Structure Model for Engineer-to-Order Production	Shingo Akasaka, Jiahua Weng, Hisashi Ohnari (Waseda University)
15:45-16:05	SU14-2 Optimal Ordering Decision of Supply Chain by Increasing the Intermediary	Ping-Hui Hsu (De Lin Institute of Technology), Hui-Ming Teng(Chihlee Institute of Technology)	SU24-2 A Hybrid MCDM Model for New Product Development in the LiFePO4 battery Industry Using FDM, ISM and FANP	Wen Chen, Li Wang (Chung Hua University)
16:05-16:25	SU14-3 Decisions for Products with Limited Production	Hui-Ming Teng(Chihlee Institute of Technology), Ping-Hui Hsu(De Lin Institute of Technology), Hui-Ming Wee(Chung Yuan Christian University)	SU24-3 Disruption Management for Complex Flow Shop Scheduling with Considering Behavior	Hong-guang Bo, Yu-tao Pan, Xin Zhang, Xiao-yan Ma (Dalian University of Technology)
17:15-19:15	Workshop Dinner (Saijo HAKUWA Hotel)			

SU14-3

Decisions for Products with Limited Production

Hui-Ming Teng(Chihlee Institute of Technolog), Ping-Hui Hsu(De Lin Institute of Technology), Hui-Ming Wee(Chung Yuan Christian University)

Producing limited quantity of certain products is an important strategy for dealing with scarcity and customer response. Consumers often consider scarce products as possessing higher values, and it triggers them to desire these products. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized. Numerical example is presented to demonstrate the model.

Full Paper

Decisions for Products with Limited Production

Hui-Ming Teng[†1], Ping-Hui Hsu[†2*] and Hui-Ming Wee[†3]

Abstract: Producing limited quantity of certain products is an important strategy for dealing with scarcity and customer response. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin. In this study, we consider a newsvendor problem for products with limited production quantity: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity. An algorithm is developed to obtain a production policy such that the expected profit is maximized.

Key words: Limited production quantity; Newsvendor; Scarcity

1. INTRODUCTION

Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior [1]. Consumers often consider scarce products as possessing higher values, and it triggers them to desire these products. Consequently, some manufacturers may design their marketing strategies by producing a limited quantity of their products. This type of marketing strategy is very common among innovative products. For example, department stores announce limited products on their promotional flyers during their anniversary sales. Customers are required to book in advance or wait in line in order to buy the limited products.

Commodity theory [2, 7] claims that "any commodity will be valued if it is unavailable". According to the theory, scarcity enhances the value (or desirability), and it gives to its possessor a sense of pride in possessing the limited product [1]. Sirgy [3] addressed the importance of scarcity in marketing strategy. Salespersons should apply such strategy while merchandising products or services; it will increase the motivation of the targeted customers to approach the promotional information. There are two strategies for price raise through scarcity: (1) direct result from the quality and symbolic interest, and (2) indirect result on quality and symbolic interest through the price. As a result, raising the prices of scarce products can make a positive impact, but also may backfire if it is not launch properly [4]. Therefore, if we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.

Recently, many industries apply the strategy of limited production through single production schedule. From our literature search, no researches have been done on the newsvendor problem to consider the limited production quantity issues. In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier. We present an algorithm to derive an optimal production quantity and selling price such that the expected profit is maximized.

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[†2] De Lin Institute of Technology
[†3] Chung Yuan Christian University

2. NOTATIONS

The following notations are used in our analysis:

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the optimal production quantity for the supplier considering limited production quantity
Q_w	the production quantity for the supplier without considering limited production quantity
Q_w^*	the optimal production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity ; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant
$p(Q)$	the selling price per unit with considering limited production quantity ; which is a function of production quantity
$C_p(Q)$	the production cost per unit; which is a function of production quantity, $c_p(Q) < p(Q)$.
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
c_a	the marketing cost
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$

3. MODELING AND ASSUMPTIONS

Throughout this study, single production of the limited product is assumed. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers. The unit production price of the product is c_p. For simplicity, the unit production cost is assumed to be constant. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q)= p_1$, the supplier will manufacture an optimal batch of Q_w^* . This is identical to the newsboy problem. The suppliers' expected profit function $E\pi$ is:

$$E\pi(Q_w) = \int_0^{Q_w} \{[p_1 - c_p]x - (c_p - s)(Q_w - x)\} f(x)dx + \int_{Q_w}^{\infty} \{[p_1 - c_p]Q_w - (x - Q_w)r\} f(x)dx. \quad (1)$$

Similar to Hadley and Whitin (1963), the suppliers' optimal production batch is:

$$F(Q_w^*) = (p_1 - c_p + r)/(p_1 - s + r), \quad (2)$$

where $F(x)$ is the CDF of x. If the supplier manages the limited production batch, then the consumers' perceived value and purchase decisions are usually influenced by the law of scarcity [17]. The unit selling price $p(Q)$ of the limited quantity products is a decreasing function of Q. However, the customer demand will decrease due to a higher selling price. That is, the random demand of the products depends on production batch, Q, because the higher production batch will decrease the selling price, while the lower selling price will increase demand. That means the PDF, $f(x)$, of the random demand x is a function of Q. The suppliers' expected profit function $E\pi$ is given as follows:

$$E\pi(Q) = \int_0^{Q} \{[p(Q) - C_p(Q)]x - [C_p(Q) - s](Q - x)\} f(x)dx + \int_{Q}^{\infty} \{[p(Q) - C_p(Q)]Q - (x - Q)r\} f(x)dx - c_a. \quad (3)$$

Our problem can be formulated as:

$$Max: E\pi(Q). \quad (4)$$

We illustrate the model by a case study.

4. AN ILLUSTRATIVE CASE STUDY

In this section, a practical selling price and probability distribution are applied to explain the results of the previous section. Since the selling price is always influenced by the limited production quantity (Wu & Hsing, [5]), the selling price per unit $p(Q)$ can therefore be assumed as:

$$p(Q) = \frac{p_2 - p_1}{\sqrt{Q}} + p_1,\ p_2 > p_1 > 0,\ Q \geq 1, \quad (5)$$

which means $p_1 < p(Q) < p_2$ and is a decreasing function of Q (Please refer to Figure 1a). For the supplier, the random demand is assumed to be uniformly distributed over the range 0 and $B(Q)$, where

$$B(Q) = \frac{bp_1}{p(Q)}, \quad (6)$$

is a function of Q with positive constant b (b is the upper bound of the selling quantity). This means that a higher selling price would decrease demand. Thus, the PDF of the supplier's demand is

$$f(x) = \frac{1}{B(Q)}. \quad (7)$$

In the same time, the production cost is always influenced by the limited production quantity, the production cost per unit

$$C_p(Q) = \frac{c_{p2} - c_{p1}}{Q} + c_{p1},\ c_{p2} > c_{p1} > 0,\ Q \geq 1, \quad (8)$$

which means $c_{p1} < C_p(Q) < c_{p2}$ and is a decreasing function of Q (Please refer to Figure 1b). Obviously, $C_p(Q) < p(Q)$.

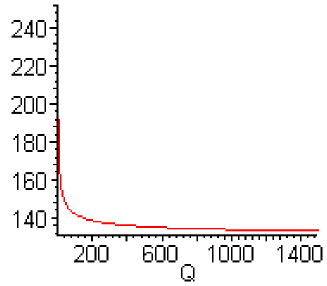

Fig. 1a. The shape of *p*(*Q*), 0<Q<1500 in example 1

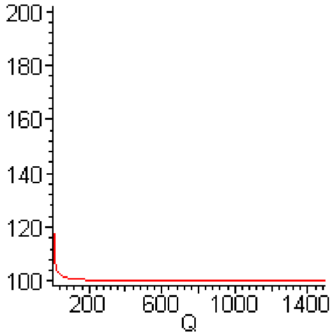

Fig. 1b. The shape of *Cp*(*Q*), 0<Q<1500 in example 1

From, $B(Q) = \dfrac{bp_1}{p(Q)}$, one has (Calculated by mathematical software Maple 8)

$$E\pi'(Q) = \int_0^Q [p'(Q)x - c_p + s]\frac{1}{B(Q)}dx + \int_Q^{B(Q)} [p'(Q)Q + p(Q) - c_p + r]\frac{1}{B(Q)}dx + B'(Q)f(B(Q))\left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}. \quad (9)$$

$$E\pi''(Q) = p''(Q)\frac{Q^2}{2B(Q)} + \left[p''(Q)Q + 2p'(Q)\right]\frac{B(Q)-Q}{B(Q)} - \left[p(Q) - s + r\right]f(Q) + \left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}\left[B'(Q)\right]^2 f'(B(Q)) + \left\{2B'(Q)\left[p'(Q)Q + p(Q) - c_p + r\right] + B''(Q)\left[p(Q)Q - c_pQ - rB(Q) + rQ\right] - r\left[B'(Q)\right]^2\right\}f(B(Q)). \quad (10)$$

From (9), it is hard to prove the concavity of $E\pi(Q)$. A numerical example is provided to illustrate the model.

Example 1. Given p_2=250, p_1=130, c_p=100, c_{p1}=100, c_{p2}=200, b=1500, s=50, r=5 and c_a=1000, then

$$E\pi(Q) = -(480558000Q^{\frac{1}{2}} - 154405544Q - 35390614Q^{\frac{3}{2}} + 9360Q^2 + 2873Q^{\frac{5}{2}} + 102960000)/[7800(12 + 13Q^{\frac{1}{2}})]$$

$$E\pi'(Q) = -(1107054000Q^{\frac{1}{2}} - 926432264Q - 82033344Q^{\frac{3}{2}} - 22891596Q^2 + 134355Q^{\frac{5}{2}} + 37349Q^3)/[3900Q(12 + 13Q^{\frac{1}{2}})^2]$$

$$E\pi''(Q) = (13284648000Q^{\frac{1}{2}} + 43175106000Q - 1424324736Q^{\frac{3}{2}} - 517035168Q^2 - 4836780Q^{\frac{5}{2}} - 3539367Q^3 - 971074Q^{\frac{7}{2}})/[7800Q^2(12 + 13Q^{\frac{1}{2}})^3].$$

For the above equation of $E\pi''(Q)$, the first and third term, (13284648000-1424324736Q) $Q^{\frac{1}{2}}$, and the second and fourth term, (43175106000-517035168Q)Q are negative since Q>84. That means that $E\pi''(Q)$ <0, therefore $E\pi(Q)$ is concave. The concavity of $E\pi(Q)$ is also illustrated in Figures 2, 3, and 4. Figure 2 presents the curve for $E\pi(Q)$; it reaches a maximum in the interval [500, 800]. Figure 4 presents the curve for $E\pi''(Q)$ showing its negative value. It shows $E\pi(Q)$ is concave on [50,1500]. Figure 3 presents the curve for $E\pi'(Q)$; it shows the root of $E\pi'(Q)$=0, and it is located in the interval [500, 1000]. Setting $E\pi'(Q)$ equals to zero, Q^*=614 is derived using Maple 8, the selling price per unit is $p(Q^*)$=\$134.8, and the optimal expected profit for the supplier is $E\pi(Q^*)$ =\$8037. When limited production quantity is not considered, Q_w^* =618 (using Eq.(2)), $E\pi(Q_w^*)$ =\$7059 (using Eq.(1)), and the percentage profit increase is $\dfrac{E\pi(Q^*)}{E\pi(Q_w^*)} - 1$ =13.9%.

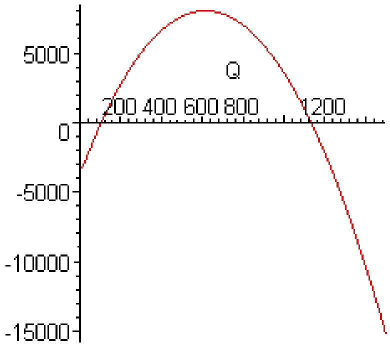

Fig. 2. The shape of $E\pi(Q)$, 0<Q<1500 in example 1

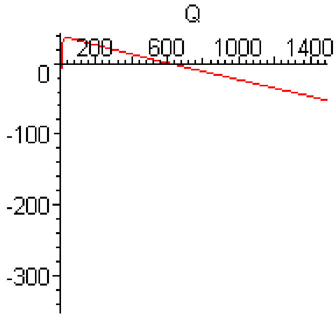

Fig. 3. The shape of $E\pi'(Q)$, 0<Q<1500 in example 1

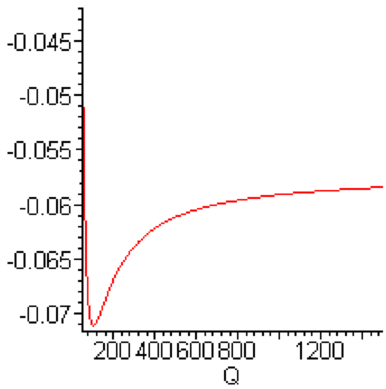

Fig. 4. The shape of $E\pi''(Q)$, 50<Q<1500 in example 1

5. CONCLUSION

Limited production strategy allows the firm to manipulate the scarcity effect [2] of the products with limited production quantity. This enables the firm to increase the product selling prices due to exclusive distribution outlets. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.

Illustrative case study and numerical example are presented to demonstrate the proposed model. The numerical example show that the percentage profit increase is fairly significant. Most of past researches focused on launch timing [5] and reciprocal effects [6]. The production and ordering quantities in this study directly influence the profit. Future researches are suggested to consider varying distribution of customer demand.

ACKNOWLEDGMENTS

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Decisions for Products with Limited Production

Author：Hui-Ming Teng
Ping-Hui Hsu
Hui-Ming Wee
Presenter： Hui-Ming Teng
Date ：2014/10/07

Agenda

1、ABSTRACT

1. Producing limited quantity of certain products is an important strategy for **dealing with scarcity and customer response**.
2. Some distribution outlet would raise prices to cover promotion expenses and to increase profit margin.
3. In this study**, we consider a newsvendor problem for products with limited production quantity**: the unit selling price, the unit production cost and customers' demand are influenced by the limited production quantity.
4. An algorithm is developed to obtain a production policy such that the expected profit is maximized.

2. INTRODUCTION(1)

- Scarcity of matter is a pervasive aspect of human life and is the fundamental precondition of economic behavior(Lynn, M. 1991)
- Commodity theory claims that "any commodity will be valued if it is unavailable".
- There are two strategies for price raise through scarcity:

  (1) direct result from the quality and symbolic interest,

  (2) indirect result on quality and symbolic interest through the price.

2. INTRODUCTION(2)

- If we combine the commodity theory and the need for uniqueness theory, we can demonstrate that customers prefer possessing scarce product to show their uniqueness, compared to possessing common and easily-available products.
- In this study, the supplier has to consider the uncertainty in customer demand. Having a good manufacturing and marketing strategy of the limited-edition products before the selling period of the product is vital to the supplier.

3.NOTATIONS(I)

$E\pi$	the expected profit for the supplier
Q	the production quantity for the supplier; decision variable
Q^*	the optimal production quantity for the supplier considering limited production quantity
Q_w	the production quantity for the supplier without considering limited production quantity
Q_w^*	the optimal production quantity for the supplier without considering limited production quantity
p_1	the selling price per unit without considering limited production quantity ; constant
p_2	the upper bound of selling price per unit when the production quantity is limited; constant

3.NOTATIONS(II)

$p(Q)$	the selling price per unit with considering limited production quantity ; which is a function of production quantity
$C_p(Q)$	the production cost per unit; which is a function of production quantity, $c_p(Q) < p(Q)$.
s	the salvage value per unit $s < c_p$
r	the shortage cost per unit; represents costs of lost goodwill
c_a	the marketing cost
x	the random demand with the PDF (Probability Density Function), $f(x)$, and CDF (Cumulative Distribution Function), $F(x)$.

4、MODELING AND ASSUMPTIONS

1. Single production of the limited product is assumed.
2. The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers.
3. The unit production price of the product is c_p. the unit production cost is assumed to be constant.
4. The unit selling price is $p(Q)$. When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s.
5. When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price $p(Q)=P_l$, the supplier will manufacture an optimal batch of Q_w^*.

Demand < Production quantity

$$E\pi(Q)=\int_0^Q\left\{\left[p(Q)-C_p(Q)\right]x-\left[C_p(Q)-s\right](Q-x)\right\}f(x)dx$$

$$+\int_Q^{\infty}\left\{\left[p(Q)-C_p(Q)\right]Q-(x-Q)r\right\}f(x)dx-c_a. \qquad (3)$$

Demand > Production quantity

Our problem can be formulated as:

$$Max: E\pi(Q). \qquad (4)$$

Since the selling price is always influenced by the limited production quantity (Wu & Hsing, [5]), the selling price per unit $p(Q)$ car

$$p(Q) = \frac{p_2}{\;}$$

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$$B(Q) = \frac{\;}{p(Q)}$$

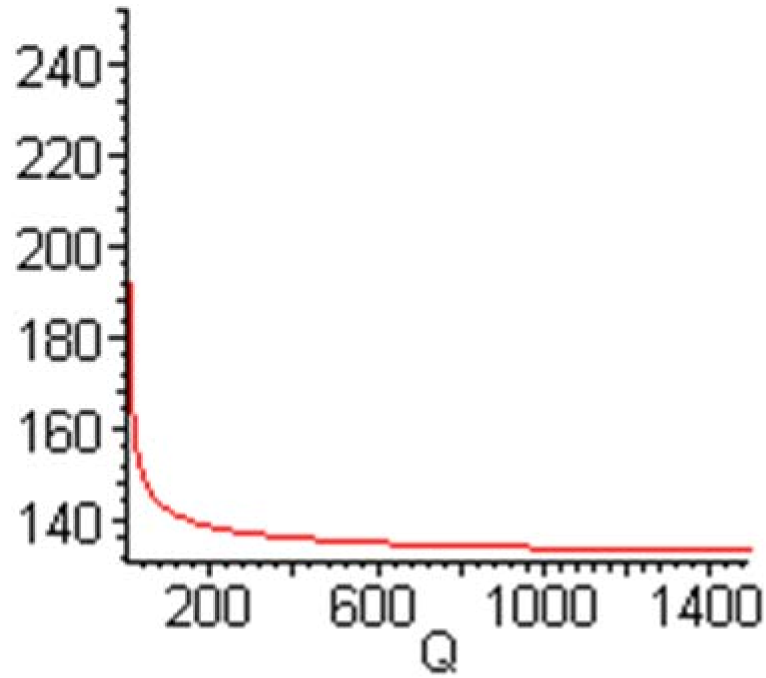

Fig. 1a. The shape of $p(Q)$, 0<Q<1500 in example 1

This means that a higher selling price would decrease de-
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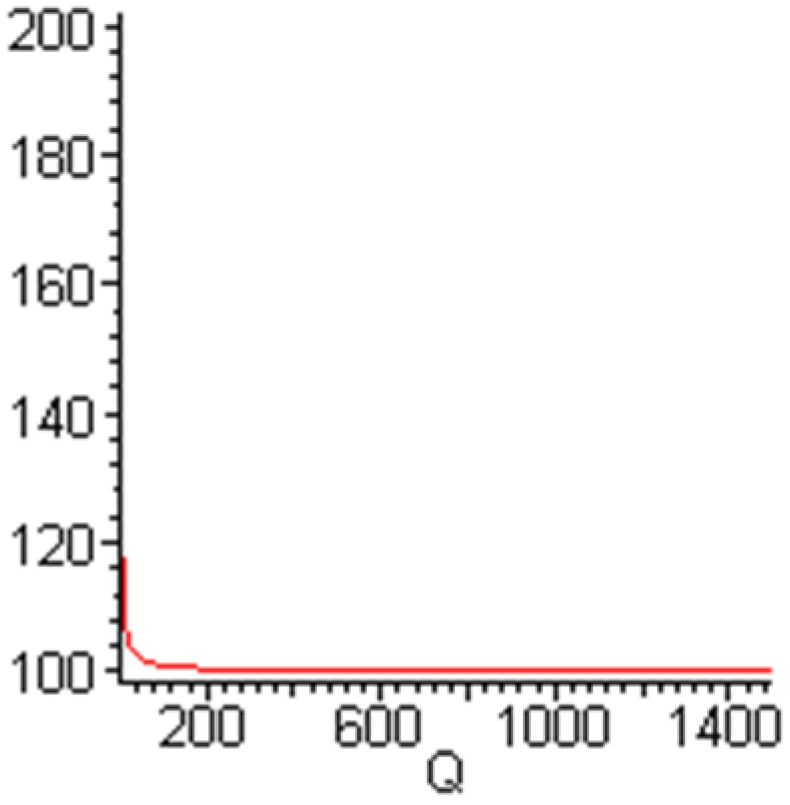

Fig. 1b. The shape of $Cp(Q)$, $0<Q<1500$ in example 1

$$E\pi'(Q) = \int_0^Q [p'(Q)x - c_p + s]\frac{1}{B(Q)}dx + \int_Q^{B(Q)} [p'(Q)Q + p(Q) - c_p + r]\frac{1}{B(Q)}dx + B'(Q)f(B(Q))\left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}. \quad (9)$$

$$E\pi''(Q) = p''(Q)\frac{Q^2}{2B(Q)} + \left[p''(Q)Q + 2p'(Q)\right]\frac{B(Q)-Q}{B(Q)} - \left[p(Q) - s + r\right]f(Q) + \left\{\left[p(Q) - c_p\right]Q - \left[B(Q) - Q\right]r\right\}\left[B'(Q)\right]^2 f'(B(Q))$$

$$+\left\{2B'(Q)\left[p'(Q)Q + p(Q) - c_p + r\right] + B''(Q)\left[p(Q)Q - c_pQ - rB(Q) + rQ\right] - r\left[B'(Q)\right]^2\right\}f(B(Q)).$$

(10)

From (9), it is hard to prove the concavity of $E\pi(Q)$. A numerical example is provided to illustrate the model.

Example 1. Given p_2=250, p_1=130, c_p=100, c_{p1}=100, c_{p2}=200, b=1500, s=50, r=5 and

c_a =1000, then

$$E\pi(Q) = -(480558000Q^{\frac{1}{2}} - 15440544Q - 3539064Q^{\frac{3}{2}} + 9360Q^{2} + 2873Q^{\frac{5}{2}} + 102960000)/[7800(12+13Q^{\frac{1}{2}})]$$

$$E\pi'(Q) = -(1107054000Q^{\frac{1}{2}} - 92643264Q - 8203334Q^{\frac{3}{2}} - 2289159 6Q^{2} + 134355Q^{\frac{5}{2}} + 37349Q^{3})/[3900Q(12+13Q^{\frac{1}{2}})^{2}]$$

$$E\pi''(Q) = (13284648000Q^{\frac{1}{2}} + 43175106000Q - 1424324736Q^{\frac{3}{2}} - 517035168Q^{2} - 4836780Q^{\frac{5}{2}} - 3539367Q^{3} - 971074Q^{\frac{7}{2}})/[7800Q^{2}(12+13Q^{\frac{1}{2}})^{3}].$$

For $E\pi''(Q)$, the first and third term,

$(13284648000\text{-}1424324736Q)Q^{\frac{1}{2}}$,

and the second and fourth term,

$(43175106000\text{-}517035168Q)Q$

are negative since $Q>84$.

That means that $E\pi''(Q)<0$, therefore $E\pi(Q)$ is concave.

The concavity of $E\pi(Q)$ is illustrated in Figures 2, 3, and 4.

Figure 2 presents the curve for $E\pi(Q)$

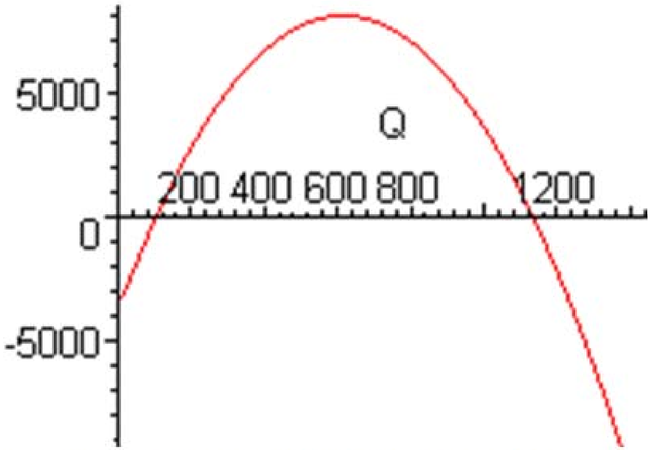

Fig. 2. The shape of $E\pi(Q)$, 0<Q<1500
in example 1

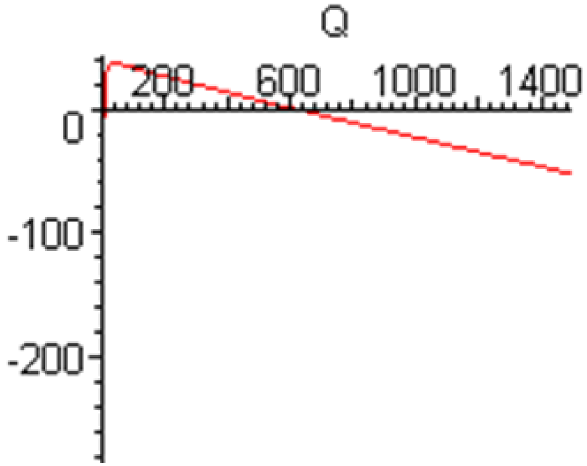

Fig. 3. The shape of $E\pi'(Q)$, 0<Q<1500
in example 1

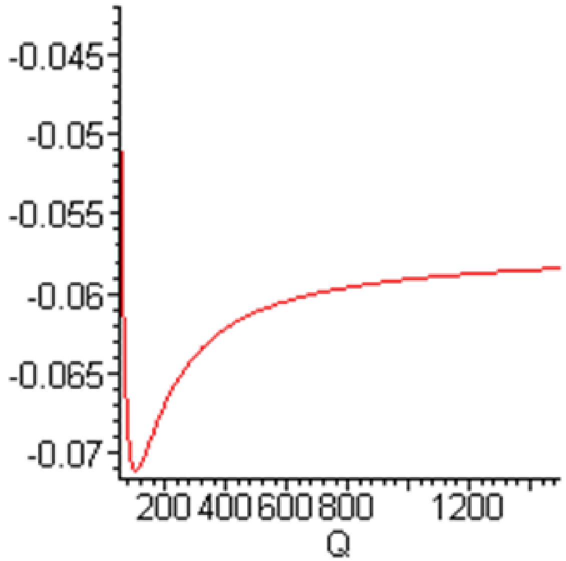

Fig. 4. The shape of $E\pi''(Q)$, 50<Q<1500
in example 1

Setting $E\pi'(Q)$ equals to zero, Q^*=614 is derived using Maple 8,

the selling price per unit is $p(Q^*)$=\$134.8,

the optimal expected profit for the supplier is $E\pi(Q^*)$ =\$8037.

When limited production quantity is not considered, Q_w^* =618 (using Eq.(2)),

$E\pi(Q_w^*)$ =\$7059 (using Eq.(1)),

and the percentage profit increase is $\frac{E\pi(Q^*)}{E\pi(Q_w^*)}-1$ =13.9%.

6.CONCLUSION(1)

1. Limited production strategy allows the firm to manipulate the scarcity effect [2] of the products with limited production quantity.
2. This enables the firm to increase the product selling prices due to exclusive distribution outlets.
3. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.
4. Illustrative case study and numerical example are presented to demonstrate the proposed model.

6.CONCLUSION(2)

5. The numerical example show that the percentage profit increase is fairly significant.
6. Most of past researches focused on launch timing [5] and reciprocal effects [6].
7. The production and ordering quantities in this study directly influence the profit. Future researches are suggested to consider varying distribution of customer demand.

Q &A


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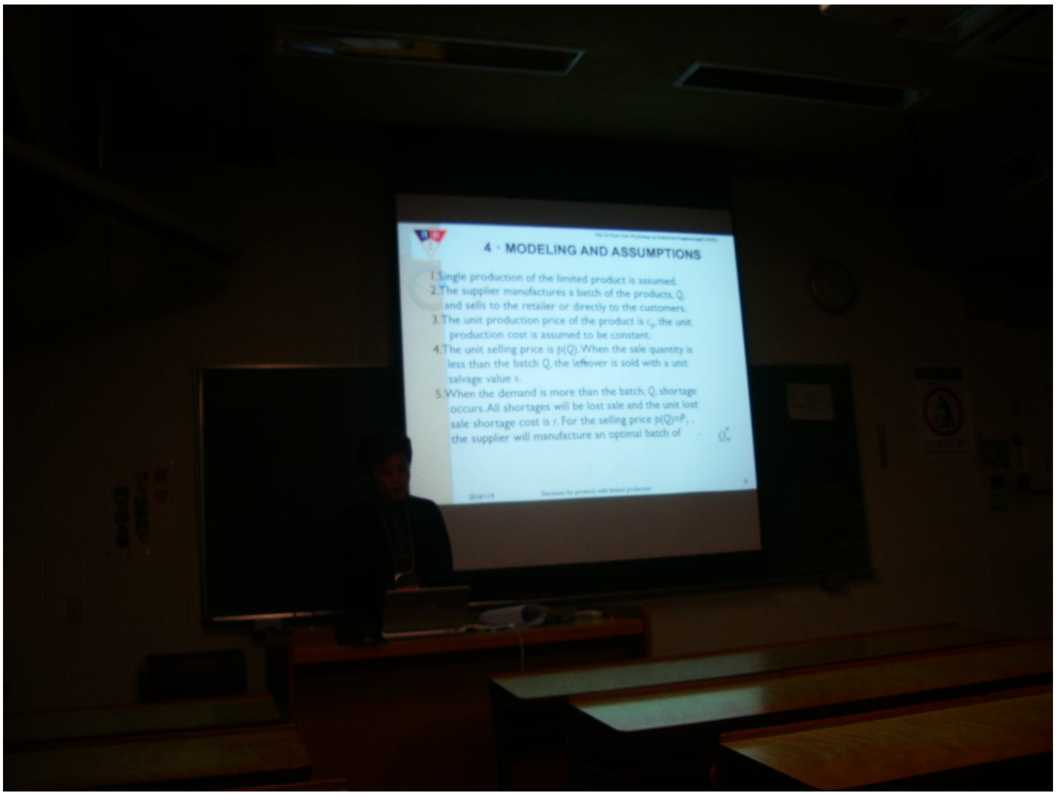
4 · MODELING AND ASSUMPTIONS
1.Single production of the limited product is assumed.
2.The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers.
3.The unit production price of the product is c, the unit production cost is assumed to be constant.
4.The unit selling price is p(Q). When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s.
5.When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price
the supplier will manufacture an optimal batch of


4 · MODELING AND ASSUMPTIONS
1.Single production of the limited product is assumed.
2.The supplier manufactures a batch of the products, Q, and sells to the retailer or directly to the customers.
3.The unit production price of the product is c_p, the unit production cost is assumed to be constant.
4.The unit selling price is p(Q). When the sale quantity is less than the batch Q, the leftover is sold with a unit salvage value s.
5.When the demand is more than the batch, Q, shortage occurs. All shortages will be lost sale and the unit lost sale shortage cost is r. For the selling price p(Q)=P_i , the supplier will manufacture an optimal batch of


広島大学
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GATE

科技部補助計畫衍生研發成果推廣資料表

日期:2015/08/24

科技部補助計畫	計畫名稱: 限量生產的行銷整合策略	
	計畫主持人: 滕慧敏	
	計畫編號: 103-2221-E-263-002-	學門領域: 生產系統規劃與管制

無研發成果推廣資料

103年度專題研究計畫研究成果彙整表

計畫主持人：滕慧敏			計畫編號：103-2221-E-263-002-				
計畫名稱：限量生產的行銷整合策略							
成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	1	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	1	1	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	無

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

科技部補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：■已發表 □未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

In this study, we derive a newsvendor problem model with limited production quantity. Limited production strategy allows the firm to manipulate the scarcity effect of the products with limited production quantity. This enables the firm to increase the product selling prices due to exclusive distribution outlets. In analyzing the system, we provide managerial insights to decision makers in planning production quantity and selling price in order to derive the optimal profit.

Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

The two numerical examples show that the percentage profit increase is fairly significant. Most of past researches focused on

launch timing [16] and reciprocal effects [17]. The production and ordering quantities in this study directly influence the

profit. For simplicity, the unit production cost is assumed to be constant. Future researches are suggested to consider variable

unit production cost, selling price and varying distribution of customer demand.