# 科技部補助專題研究計畫成果報告 期末報告

## 不完美生產過程檢驗設備策略下之最佳檢驗站數量及製造量

計畫類別:個別型計畫

計 畫 編 號 : MOST 105-2221-E-263-001-

執 行 期 間 : 105年08月01日至106年07月31日

執 行 單 位 : 致理學校財團法人致理科技大學企業管理系

計畫主持人: 滕慧敏

計畫參與人員: 大專生-兼任助理:蔡季芸

報告附件:出席國際學術會議心得報告

中華民國 106 年 10 月 26 日

中 文 摘 要 : 品質要求已是現代企業的核心競爭力。為了維持產品的品質,企業 必須付出相對的成本,因此如何取捨品質與成本的平衡,對管理者 相當重要。本研究建構了數學模式,以求出最佳訂購量及最佳設備 檢驗數量,使總利潤及最佳製造批量為最佳,研究中並藉由數值範 例進行驗證;文中並進行敏感性分析。

中文關鍵詞:檢驗設備,不完美製造過程,品質成本

英文摘要: The trading-off between quality requirements and quality costs are an important issue. This study derives an optimal order quantity and the optimal number of screening equipment such that the total profit per manufactured batch is maximized. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

英文關鍵詞: Screening equipment, Imperfect production process, Quality costs

### 科技部補助專題研究計畫成果報告

### (□期中進度報告/☑期末報告)

### 不完美生產過程檢驗設備策略下之最佳檢驗站數量及製造量

計畫類別: ☑個別型計畫 □整合型計畫

計畫編號: MOST 105-2221-E-263-001-

執行期間: 2016 年08 月01 日至2017 年07 月31 日

執行機構及系所:致理學校財團法人致理科技大學企業管理系

計畫主持人: 滕慧敏

共同主持人:大專生-兼任助理人員:蔡季芸

計畫參與人員:

本計畫除繳交成果報告外,另含下列出國報告,共乙份:

- □執行國際合作與移地研究心得報告
- □出席國際學術會議心得報告
- □出國參訪及考察心得報告

中 華 民 國 106 年 10 月 31 日

### 中文摘要:

品質要求已是現代企業的核心競爭力。為了維持產品的品質,企業必須付出相對的成本, 因此如何取捨品質與成本的平衡,對管理者相當重要。本研究建構了數學模式,以求出最佳訂 購量及最佳設備檢驗數量,使總利潤及最佳製造批量為最佳,研究中並藉由數值範例進行驗 證;文中並進行敏感性分析。

關鍵字:檢驗設備;不完美製造過程;品質成本

### **ABSTRACT**

The trading-off between quality requirements and quality costs are an important issue. This study derives an optimal order quantity and the optimal number of screening equipment such that the total profit per manufactured batch is maximized. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

Keywords: Screening equipment, Imperfect production process, Quality costs

# OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS

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### **ABSTRACT**

The trading-off between quality requirements and quality costs are an important issue. This study derives an optimal order quantity and the optimal number of screening equipment such that the total profit per manufactured batch is maximized. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

**Keywords:** Screening equipment, Imperfect production process, Quality costs

### 1 INTRODUCTION

In striving to manufacture quality products, manufacturers not only have to consider manufacturing costs, but also must maintain the quality requirements to remain competitive. However, to improve upon the product's quality it is necessary to reduce defective items within the design, production and screening process.

The manufacturing process of Hi-tech products usually requires the need for many workstations. Each semi-finished product in the workstation is the raw material of the next workstation • If the semi-finished product is defective and is not repaired on the workstation, then it is rendered a defective item after moving to the next station. • As a result, the screening process of workstations plays an important role in maintaining quality control. However, the screening process requires relevant equipment and trained professionals. For the sake of maintaining good quality, the related cost is needed. Therefore, the trade-off between the quality and cost is an important factor for managers.

In practice, managers must first count the percentage of defective products in each workstation and subsequently determines the necessity for investing in screening equipment. The objective of the trade-off between gains from good items and losses from defective items to maximize profit.

Rosenblatt and Lee (1986) were early researchers who considered defective items and imperfect quality production processes. Salameh and Jaber (2000) displayed an inventory model which accounted for imperfect items using the EPQ/EOQ formulae. Eroglu and Ozdemir (2007) developed an economic order quantity model with defective items and shortages. Sana (2011) presented an

integrated production-inventory model for supplier, manufacturer and retailer supply chain, considering perfect and imperfect quality items. Hsu and Hsu (2013) developed an integrated inventory model for vendor–buyer coordination under an imperfect production process.

Many companies promote quality as the central customer value and consider it to be a critical success factor for achieving competitiveness. Any serious attempt to improve quality must take into account the costs associated with achieving quality since the objective is to meet customer requirements at the lowest cost (Schiffauerova and Thomson, 2006). Quality costs are the costs incurred in the design, implementation, operation and maintenance of a quality management system, the cost of resources committed to continuous improvement, the costs of system, product and service failures, and all other necessary costs and non-value added activities required to achieve a quality product or service (Dale and Plunkett, 1995). Chiu and Su (2010) considered the quality cost, the time-value of money, and the exponential process quality improvement function in constructing a new total cost model to optimize the production period, and initial investment in process improvement so as to minimize total cost.

Most of the past research have discussed the inventory problems of imperfect items. However, to the best of our knowledge, little research had been found on improving the imperfect rate of items. This study considers an inventory model of ordering raw material one-time for the production to meet the quantity and quality of the orders. An algorithm is presented to derive an optimal order quantity for the raw material and the number of screening equipment required such that the total profit per manufactured batch is maximized.

### 2 ASSUMPTIONS AND NOTATION

- (1). Defective units are directly returned for re-manufacturing in the same workstation as the screening equipment is set.
- (2). Shortage is not allowed.
- (3). Single manufacturer and single retailer are considered.
- (4). The capacity of the warehouse is unlimited.

### The following notations are used:

- p defective percentage in per manufactured workstation
- $\delta$  unit manufacture's compensation cost for less achieve percentage of retailer's order quantity
- N total number of workstation, integer
- s salvage per imperfect manufactured unit
- r revenue per perfect manufactured unit  $(0 \le r \le 1)$
- $\beta$  manufacturer 's delivery percentage
- Q manufacture's order quantity for the raw material
- k number of workstation with the screening equipment, integer, variable
- $C_s$  screening cost containing the equipment, (\$ / screening equipment)

- $C_m$  manufacturing cost per unit containing material ( \$ / unit )
- $C_o$  ordering cost (\$ / order)
- $Q_o$  retailer's ordering quantity for the manufactured unit
- TP total profit per manufacturing batch

### 3 Model MODEL DEVELOPMENT

In this study, a supply chain with a manufacturer and a retailer is assumed. The retailer obtains the products from the manufacturer for sale to the customers.

Assume that the manufacturer producing the items from raw material into a finished product needs n workstations, and the semi-finished products outputs from each workstation with defective percentage of p. Assume that every screened imperfect item can be served as a good item after being repaired in the same workstation, and passed through to the next workstation. If the workstation is not set up with the screening equipment, the imperfect items in the workstation are still regarded as defective after passing out of the next station, and cannot be returned for repair. The imperfect items can be let out for recovery with salvage value s. Assume that the manufacturing cost per unit containing material is  $C_m$ . Assume that the total number of workstation is N, while there are only k of the screening equipment are set in the workstation for the sake of reducing the screening costs (K < n-1). Although, less screening equipments can lead to lowered costs, more defective products are produced. Based on maximizing the total profit per manufacturing batch, TP, how to decide the order quantity for the raw material, Q, by the manufacturer when the retailer's ordering quantity for the manufactured unit is Qo?

From the statement above, one has

The total profit per manufacturing batch, TP is as follows:

TP =Revenue of perfect items from per manufactured batch

- + Salvage of imperfect items from per manufactured batch
- Total manufacturing cost -Ordering cost-Total screening cost

The constraint is that the quantity of the perfect items from per manufactured batch is the retailer's ordering quantity,  $Q_o$ . One has

$$TP(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s.$$

(1)

The following two cases consider the systems with the constant defective percentage and variable defective percentage.

Case 1: When the achieve percentage of the retailer's ordering quantity is 100%.

Our problem can be formulated as:

Maximize: TP(k, Q)

Subject to:  $Q(1-p)^{N-k} = Q_o, k \le N.$ 

(2)

From (2),

$$Q = \frac{Q_o}{\left(1 - p\right)^{N - k}}.$$

(3)

Substitute Eq. (3) into (1), TP(k, Q) can be transformed as

$$TPx(k) = rQ_o + \frac{sQ_o[1 - (1 - p)^{N - k}]}{(1 - p)^{N - k}} - \frac{c_mQ_o}{(1 - p)^{N - k}} - c_o - c_s k.$$
(4)

That is, our problem can be formulated as:

Maximize: TPx(k),  $k \le N$ .

(5)

Since

$$TPx'(k) = \frac{sQ_o[1 - (1-p)^{N-k}]\ln(1-p)}{(1-p)^{N-k}} + sQ_o\ln(1-p) - \frac{c_mQ_o\ln(1-p)}{(1-p)^{N-k}} - c_s.$$

(6)

$$TPx''(k) = \frac{sQ_o[1 - (1 - p)^{N - k}]\ln^2(1 - p)}{(1 - p)^{N - k}} + sQ_o \ln^2(1 - p) - \frac{c_mQ_o \ln^2(1 - p)}{(1 - p)^{N - k}}.$$

(7)

Simplify (7), one has

$$TPx''(k) = Q_o \ln^2(1-p)(1-p)^{-(N-k)}(s-c_m) < 0.$$

(8)

(Note:  $s < c_m$ ) Therefore, which leads to the function TPx(k) being strictly concave.

Set TPx'(k) = 0, one has

$$k = \frac{N \ln(1-p) - \ln(\frac{Q_o \ln(1-p)(s-c_m)}{c_s})}{\ln(1-p)}.$$

(9)

Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for TP at  $k = k^*$ ,  $Q = Q^*$  is

$$\Delta TP(k^*, Q^*) > 0 > \Delta TP(k^* - 1, Q^*)$$

(10)

where

$$\Delta TP(k^*, Q^*) = \Delta TP(k^* + 1, Q^*) - \Delta TP(k^*, Q^*)$$
.

(11)

### Example 1.

Assuming N=30, p=0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_s=550$ , and  $Q_o=10000$ .

Then from (9 and 10), k=21.016, because TP(21,Q)=133416 and TP(22,Q)=133414, therefore  $k^*=21$ , with  $Q^*=10947$ , and  $TP(k^*,Q^*)=133416$ .

### Case 2: When the achieve percentage of the retailer's ordering quantity is less than 100%.

In practice, short delivery always happen. Compensation mechanism came into being. In this case, the manufacturer can only set the delivery  $\beta\%$  in the last resort. Then the retailer needs to make compensation  $0.01\beta\delta$  for the manufacturer. When the achieve percentage of the retailer's ordering quantity is  $\beta\%$ , the total profit per manufacturing batch is

$$TP_1(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s - 0.01(100-\beta)\delta.$$
(12)

Our problem can be formulated as:

Maximize:  $TP_1(k,Q)$ 

Subject to: 
$$Q(1-p)^{N-k} = 0.01\beta Q_o, k \le N.$$
 (13)

From (3),  $TP_1(k,Q)$  can be transformed as

$$TP_{1}x(k) = r0.01\beta Q_{o} + \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}]}{(1 - p)^{N - k}} - \frac{c_{m}0.01\beta Q_{o}}{(1 - p)^{N - k}} - c_{o} - c_{s}k - 0.01(100 - \beta)\delta.$$
(14)

That is, our problem can be formulated as:

Maximize:  $TP_1x(k)$ ,  $k \le N$ .

(15)

Since 
$$TP_1x'(k) = \frac{s0.01\beta Q_o[1-(1-p)^{N-k}]\ln(1-p)}{(1-p)^{N-k}} + s0.01\beta Q_o \ln(1-p) - \frac{c_m 0.01\beta Q_o \ln(1-p)}{(1-p)^{N-k}} - c_s$$
.

(16)

$$TP_{1}x''(k) = \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}]\ln^{2}(1 - p)}{(1 - p)^{N - k}} + s0.01\beta Q_{o}\ln^{2}(1 - p) - \frac{c_{m}0.01\beta Q_{o}\ln^{2}(1 - p)}{(1 - p)^{N - k}}.$$
 (17)

Simplify (17), one has

$$TP_1 x''(k) = 0.01 \beta Q_o \ln^2 (1-p)(1-p)^{-(N-k)} (s-c_m) < 0.$$
(18)

(Note:  $s < c_m$ ) Therefore, which leads to the function  $TP_1x(k)$  being strictly concave.

Set  $TP_1x'(k) = 0$ , one has

$$k = \frac{N \ln(1-p) - \ln(\frac{0.01\beta Q_o \ln(1-p)(s-c_m)}{c_s})}{\ln(1-p)}$$
(19)

Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for  $TP_1 x$  at  $k = k^*$ ,  $Q = Q^*$  is

$$\Delta TP_1(k^*, Q^*) > 0 > \Delta TP_1(k^* - 1, Q^*)$$
(20)

Example 2.

Assuming N=30, p=0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_s=550$ ,  $Q_o=10000$ ,  $\delta=2000$ , and  $\beta=98$ . Then from (19 and 20),

$$k^*=19$$
, with  $Q^*=10946$ , and  $TP(k^*,Q^*)=$128562$ .

### **4 CONSLUSION**

Nowadays, quality requirement is the core competition in business. For the sake of maintaining good quality, the related cost is needed. However, it is important for managers to determine the trade-off

between quality and cost. In this study, we derive an optimal order quantity for the raw material and number of screening equipment such that the total profit per manufactured batch is maximized. In analyzing the system, we provide managerial insights to decision makers for planning the optimal number of screening equipment for reducing the quality cost. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

### **5 BIBLIOGRAPHY**

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# Agenda

- Introduction
- Assumptions and Notation
- Model Development
- Conclusion
- Reference

OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS

**OPTIMAL MANUFACTURING QUANTITY FOR** 

**SCREENING EQUIPMENT POLICY WITH IMPERFECT** 

**PRODUCTION PROCESS** 

**Presenter: Hui-Ming Teng** 

Ping-Hui Hsu

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#### 1 INTRODUCTION

- In striving to manufacture quality products, manufacturers have to consider:
  - (1) manufacturing costs
  - (2) maintain the quality requirements
- To improve upon the product's quality it is necessary to reduce defective items within the design, production and screening process.
- The screening process of workstations plays an important role in maintaining quality control
- The screening process requires relevant equipment and trained professionals.
- For the sake of maintaining good quality, the related cost is needed





### 1 INTRODUCTION

- In practice, managers must first count the percentage of defective products in each workstation and subsequently determines the necessity for investing in screening equipment
- The objective of the trade-off between gains from good items and losses from defective items to maximize profit.





#### 1 INTRODUCTION

- Most of the past research have discussed the inventory problems of imperfect items. However, to the best of our knowledge, little research had been found on improving the imperfect rate of items.
- This study considers an inventory model of ordering raw material one-time for the production to meet the quantity and quality of the orders.
- An algorithm is presented to derive an optimal order quantity for the raw material and the number of screening equipment required such that the total profit per manufactured batch is maximized.

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### 2. Assumptions and Notation

### 2-1. Assumptions

- Defective units are directly returned for re-manufacturing in the same workstation as the screening equipment is set.
- Shortage is not allowed.
- Single manufacturer and single retailer are considered.
- The capacity of the warehouse is unlimited.

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### 2. Assumptions and Notation

- p defective percentage in per manufactured workstation
- $\delta$  unit manufacture's compensation cost for less achieve percentage of retailer's order quantity
- N total number of workstation, integer
- s salvage per imperfect manufactured unit
- r revenue per perfect manufactured unit (0 < r < 1)
- $\beta$  manufacturer 's delivery percentage
- Q manufacture's order quantity for the raw material
- k number of workstation with the screening equipment, integer, variable
- $C_s$  screening cost containing the equipment, ( \$ / screening equipment)
- $C_m$  manufacturing cost per unit containing material (\$ / unit)
- Co ordering cost (\$ / order)
- $oldsymbol{\mathcal{Q}}_o$  retailer's ordering quantity for the manufactured unit
- TP total profit per manufacturing batch





### 3 MODEL DEVELOPMENT

- In this study, a supply chain with a manufacturer and a retailer is assumed. The retailer obtains the products from the manufacturer for sale to the customers.
- Assume that the manufacturer producing the items from raw material into a finished product needs n workstations, and the semi-finished products outputs from each workstation with defective percentage of p.
- Assume that every screened imperfect item can be served as a good item after being repaired in the same workstation, and passed through to the next workstation
- If the workstation is not set up with the screening equipment, the imperfect items in the workstation are still regarded as defective after passing out of the next station, and cannot be returned for repair.





### 3 MODEL DEVELOPMENT

- The imperfect items can be let out for recovery with salvage value s.
- Assume that the manufacturing cost per unit containing material is  $C_m$ .
- Assume that the total number of workstation is N, while there are only k of the screening equipment are set in the workstation for the sake of reducing the screening costs (K<n-1).</li>
- Although, less screening equipments can lead to lowered costs, more defective products are produced.
- Based on maximizing the total profit per manufacturing batch, TP, how to decide the order quantity for the raw material, Q, by the manufacturer when the retailer's ordering quantity for the manufactured unit is Qo?

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• The total profit per manufacturing batch, *TP* is as follows:

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- + Salvage of imperfect items from per manufactured batch
- Total manufacturing cost
- Ordering cost
- -Total screening cost
- The constraint is that the quantity of the perfect items from per manufactured batch is the retailer's ordering quantity, Q<sub>o</sub>.
- One has

$$TP(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s.$$
 (1)

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Case 1: When the achieve percentage of the retailer's ordering quantity is 100%. Our problem can be formulated as:

Maximize: TP(k, Q)

Subject to: 
$$Q(1-p)^{N-k} = Q_o, k \le N.$$
 (2)

From (2),

$$Q = \frac{Q_o}{(1-p)^{N-k}}.$$
 (3)

Substitute Eq. (3) into (1), TP(k, Q) can be transformed as-

$$TPx(k) = rQ_o + \frac{sQ_o[1 - (1 - p)^{N - k}]}{(1 - p)^{N - k}} - \frac{c_mQ_o}{(1 - p)^{N - k}} - c_o - c_s k. \tag{4}$$

That is, our problem can be formulated as:

Maximize: 
$$TPx(k)$$
,  $k \le N$ . (5)





Since

$$TPx'(k) = \frac{sQ_o[1 - (1 - p)^{N - k}]\ln(1 - p)}{(1 - p)^{N - k}} + sQ_o\ln(1 - p) - \frac{c_mQ_o\ln(1 - p)}{(1 - p)^{N - k}} - c_s.$$
(6)

$$TPx''(k) = \frac{sQ_o[1 - (1-p)^{N-k}]\ln^2(1-p)}{(1-p)^{N-k}} + sQ_o \ln^2(1-p) - \frac{c_mQ_o \ln^2(1-p)}{(1-p)^{N-k}}.$$
 (7)

Simplify (7), one hase

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$$TPx''(k) = Q_o \ln^2(1-p)(1-p)^{-(N-k)}(s-c_m) < 0.$$
 (8)

(Note:  $s < c_m$ ) Therefore, which leads to the function TPx(k) being strictly concave.  $\varphi$ Set TPx'(k) = 0, one has  $\varphi$ 

$$k = \frac{N \ln(1-p) - \ln(\frac{Q_O \ln(1-p)(s-c_m)}{c_S})}{\ln(1-p)}.$$
(9)





Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for TP at  $k = k^*$ ,  $Q = Q^*$  is

$$\Delta TP(k^*, Q^*) > 0 > \Delta TP(k^* - 1, Q^*)$$
 (10)

where

$$\Delta TP(k^*, Q^*) = \Delta TP(k^* + 1, Q^*) - \Delta TP(k^*, Q^*),$$
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#### Example 1.

Assuming N=30, p=0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_s=550$ , and  $Q_o=10000$ . Then from (9 and 10), k=21.016, because TP(21,Q)=133416 and TP(22,Q)=133414, therefore  $k^*=21$ , with  $Q^*=10947$ , and  $TP(k^*,Q^*)=133416$ .

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# Case 2: When the achieve percentage of the retailer's ordering quantity is less than

In practice, short delivery always happen. Compensation mechanism came into being. In this case, the manufacturer can only set the delivery  $\beta\%$  in the last resort. Then the retailer needs to make compensation  $0.01\beta\delta$  for the manufacturer. When the achieve percentage of the retailer's ordering quantity is  $\beta\%$ , the total profit per manufacturing batch is

$$TP_1(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s - 0.01(100 - \beta)\delta.$$
 (12)

Our problem can be formulated as:

Maximize:  $TP_1(k,Q)$ 

Subject to: 
$$Q(1-p)^{N-k} = 0.01 \beta Q_o, k \le N.$$
 (13)

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From (3),  $TP_1(k,Q)$  can be transformed as

$$TP_1x(k) = r0.01\beta Q_o + \frac{s0.01\beta Q_o[1 - (1-p)^{N-k}]}{(1-p)^{N-k}} - \frac{c_m 0.01\beta Q_o}{(1-p)^{N-k}} - c_o - c_s k - 0.01(100 - \beta)\delta. \quad (14)$$

That is, our problem can be formulated as:4

$$Maximize:_{\sim} TP_1x(k), k \le N.$$
 (15)

Since

$$TP_{1}x'(k) = \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}]\ln(1 - p)}{(1 - p)^{N - k}} + s0.01\beta Q_{o}\ln(1 - p) - \frac{c_{m}0.01\beta Q_{o}\ln(1 - p)}{(1 - p)^{N - k}} - c_{s}. \quad (16)$$

$$TP_{1}x''(k) = \frac{s0.01\beta Q_{o}\left[1 - (1 - p)^{N - k}\right]\ln^{2}(1 - p)}{(1 - p)^{N - k}} + s0.01\beta Q_{o}\ln^{2}(1 - p) - \frac{c_{m}0.01\beta Q_{o}\ln^{2}(1 - p)}{(1 - p)^{N - k}}. \quad (17)$$





$$TP_{1}x''(k) = \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}] \ln^{2}(1 - p)}{(1 - p)^{N - k}} + s0.01\beta Q_{o} \ln^{2}(1 - p) - \frac{c_{m}0.01\beta Q_{o} \ln^{2}(1 - p)}{(1 - p)^{N - k}}. \quad (17)$$

Simplify (17), one has

$$TP_1 x''(k) = 0.01 \beta Q_o \ln^2 (1-p)(1-p)^{-(N-k)} (s-c_m) < 0.$$
 (18)

(Note:  $s < c_m$ ) Therefore, which leads to the function  $TP_1x(k)$  being strictly concave.  $P_1x(k) = 0$ , one has

$$k = \frac{N\ln(1-p) - \ln(\frac{0.01\beta Q_o \ln(1-p)(s-c_m)}{c_s})}{\ln(1-p)}$$
(19)





Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for  $TP_1 x$  at  $k=k^*$ ,  $Q=Q^*$  is

$$\Delta TP_1(k^*, Q^*) > 0 > \Delta TP_1(k^* - 1, Q^*)$$
 (20)

### Example 2.

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Assuming N=30, p = 0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_d=550$ ,  $Q_o=10000$ ,  $\delta=2000$ , and  $\beta=98$ . Then from (19 and 20),

$$k^*=19$$
, with  $Q^*=10946$ , and  $TP(k^*,Q^*)=$128562$ .



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### **4 CONSLUSION**

- Quality requirement is the core competition in business.
   For the sake of maintaining good quality, the related cost is needed.
- In this study, we derive an optimal order quantity for the raw material and number of screening equipment such that the total profit per manufactured batch is maximized.
- In analyzing the system, we provide managerial insights to decision makers for planning the optimal number of screening equipment for reducing the quality cost.
- Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

2017/10/26

OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS

18

### 科技部補助專題研究計畫出席國際學術會議心得報告

日期:106年5月3日

計畫編號	MOST 105-2221-E-263-001				
計畫名稱	不完美生產過程檢驗設備策略下之最佳檢驗站數量及製造量				
出國人員 姓名	滕慧敏	服務機構 及職稱	致理科技大學企業管理系		
會議時間	106年4月27日至 106年4月28日	會議地點	捷克,布拉格		
會議名稱	(中文) 第 20 屆經濟和社會發展國際科學研討會,布拉格 (英文) The 20th International Scientific Conference on Economic and Social Development, Prague				
發表題目	OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS				

### 一、 參加會議經過

本次研討會由 International Scientific Conference, Economic and Social Development 於 2017 年 4 月 27 日 至 4 月 28 日在捷克,布拉格 Oldtown Hall 舉行.於 4 月 27 日 14:30 發表,研討會收錄 68 篇論文, 7 場次論文發表。

### 二、 與會心得

本次研討會所討論的主題為經濟和社會發展問題。與會者大多為東歐及少數亞太等知名學者專家。 本人此次發表之題目為:OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS

. 大意為:品質要求已是現代企業的核心競爭力。為了維持產品的品質,企業必須付出相對的成本,因此如何取捨品質與成本的平衡,對管理者相當重要。本研究建構了數學模式,以求出最佳訂購量及最佳設備檢驗數量,使總利潤及最佳製造批量為最佳,研究中並藉由數值範例進行驗證;文中並進行敏感性分析。

### 三、 發表論文全文或摘要

# OPTIMAL MANUFACTURING QUANTITY FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS

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### **ABSTRACT**

The trading-off between quality requirements and quality costs are an important issue. This study derives an optimal order quantity and the optimal number of screening equipment such that the total profit per manufactured batch is maximized. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

Keywords: Screening equipment, Imperfect production process, Quality costs

### 1 INTRODUCTION

In striving to manufacture quality products, manufacturers not only have to consider manufacturing costs, but also must maintain the quality requirements to remain competitive. However, to improve upon the product's quality it is necessary to reduce defective items within the design, production and screening process.

The manufacturing process of Hi-tech products usually requires the need for many workstations. Each semi-finished product in the workstation is the raw material of the next workstation of If the semi-finished product is defective and is not repaired on the workstation, then it is rendered a defective item after moving to the next station. As a result, the screening process of workstations plays an important role in maintaining quality control. However, the screening process requires relevant equipment and trained professionals. For the sake of maintaining good quality, the related cost is needed. Therefore, the trade-off between the quality and cost is an important factor for managers.

In practice, managers must first count the percentage of defective products in each workstation and subsequently determines the necessity for investing in screening equipment. The objective of the trade-off between gains from good items and losses from defective items to maximize profit.

Rosenblatt and Lee (1986) were early researchers who considered defective items and imperfect quality production processes. Salameh and Jaber (2000) displayed an inventory model which accounted for imperfect items using the EPQ/EOQ formulae. Eroglu and Ozdemir (2007) developed an economic order quantity model with defective items and shortages. Sana (2011) presented an integrated production-inventory model for supplier, manufacturer and retailer supply chain, considering perfect and imperfect quality items. Hsu and Hsu (2013) developed an integrated inventory model for vendor—buyer coordination under an imperfect production process.

Many companies promote quality as the central customer value and consider it to be a critical success factor for achieving competitiveness. Any serious attempt to improve quality must take into account the costs associated with achieving quality since the objective is to meet customer requirements at the lowest cost (Schiffauerova and Thomson, 2006). Quality costs are the costs incurred in the design, implementation,

operation and maintenance of a quality management system, the cost of resources committed to continuous improvement, the costs of system, product and service failures, and all other necessary costs and non-value added activities required to achieve a quality product or service (Dale and Plunkett, 1995). Chiu and Su (2010) considered the quality cost, the time-value of money, and the exponential process quality improvement function in constructing a new total cost model to optimize the production period, and initial investment in process improvement so as to minimize total cost.

Most of the past research have discussed the inventory problems of imperfect items. However, to the best of our knowledge, little research had been found on improving the imperfect rate of items. This study considers an inventory model of ordering raw material one-time for the production to meet the quantity and quality of the orders. An algorithm is presented to derive an optimal order quantity for the raw material and the number of screening equipment required such that the total profit per manufactured batch is maximized.

### 2 ASSUMPTIONS AND NOTATION

- (1). Defective units are directly returned for re-manufacturing in the same workstation as the screening equipment is set.
- (2). Shortage is not allowed.
- (3). Single manufacturer and single retailer are considered.
- (4). The capacity of the warehouse is unlimited.

The following notations are used:

- p defective percentage in per manufactured workstation
- $\delta$  unit manufacture's compensation cost for less achieve percentage of retailer's order quantity
- N total number of workstation, integer
- s salvage per imperfect manufactured unit
- r revenue per perfect manufactured unit  $(0 \le r \le 1)$
- $\beta$  manufacturer 's delivery percentage
- Q manufacture's order quantity for the raw material
- k number of workstation with the screening equipment, integer, variable
- $C_s$  screening cost containing the equipment, (\$\\$ / screening equipment)
- $C_m$  manufacturing cost per unit containing material ( \$ / unit )
- $C_o$  ordering cost ( \$ / order)
- $Q_o$  retailer's ordering quantity for the manufactured unit
- TP total profit per manufacturing batch

### 3 Model MODEL DEVELOPMENT

In this study, a supply chain with a manufacturer and a retailer is assumed. The retailer obtains the products from the manufacturer for sale to the customers.

Assume that the manufacturer producing the items from raw material into a finished product needs n workstations, and the semi-finished products outputs from each workstation with defective percentage of p. Assume that every screened imperfect item can be served as a good item after being repaired in the same workstation, and passed through to the next workstation. If the workstation is not set up with the screening equipment, the imperfect items in the workstation are still regarded as defective after passing out of the next station, and cannot be returned for repair. The imperfect items can be let out for recovery with salvage value s.

Assume that the manufacturing cost per unit containing material is  $C_m$ . Assume that the total number of workstation is N, while there are only k of the screening equipment are set in the workstation for the sake of reducing the screening costs (K < n-1). Although, less screening equipments can lead to lowered costs, more defective products are produced. Based on maximizing the total profit per manufacturing batch, TP, how to decide the order quantity for the raw material, Q, by the manufacturer when the retailer's ordering quantity for the manufactured unit is Qo?

From the statement above, one has

The total profit per manufacturing batch, TP is as follows:

TP = Revenue of perfect items from per manufactured batch

- + Salvage of imperfect items from per manufactured batch
- Total manufacturing cost -Ordering cost-Total screening cost

The constraint is that the quantity of the perfect items from per manufactured batch is the retailer's ordering quantity,  $Q_o$ . One has

$$TP(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s.$$
 (1)

The following two cases consider the systems with the constant defective percentage and variable defective percentage.

### Case 1: When the achieve percentage of the retailer's ordering quantity is 100%.

Our problem can be formulated as:

Maximize: TP(k, Q)

Subject to: 
$$Q(1-p)^{N-k} = Q_o, k \le N.$$
 (2)

From (2),

$$Q = \frac{Q_o}{\left(1 - p\right)^{N - k}}.\tag{3}$$

Substitute Eq. (3) into (1), TP(k, Q) can be transformed as

$$TPx(k) = rQ_o + \frac{sQ_o[1 - (1 - p)^{N - k}]}{(1 - p)^{N - k}} - \frac{c_m Q_o}{(1 - p)^{N - k}} - c_o - c_s k. \tag{4}$$

That is, our problem can be formulated as:

Maximize: 
$$TPx(k), k \le N$$
. (5)

Since

$$TPx'(k) = \frac{sQ_o[1 - (1 - p)^{N - k}]\ln(1 - p)}{(1 - p)^{N - k}} + sQ_o\ln(1 - p) - \frac{c_mQ_o\ln(1 - p)}{(1 - p)^{N - k}} - c_s.$$
(6)

$$TPx''(k) = \frac{sQ_o[1 - (1 - p)^{N - k}]\ln^2(1 - p)}{(1 - p)^{N - k}} + sQ_o\ln^2(1 - p) - \frac{c_mQ_o\ln^2(1 - p)}{(1 - p)^{N - k}}.$$
 (7)

Simplify (7), one has

$$TPx''(k) = Q_0 \ln^2(1-p)(1-p)^{-(N-k)}(s-c_m) < 0.$$
 (8)

(Note:  $s < c_m$ ) Therefore, which leads to the function TPx(k) being strictly concave.

Set TPx'(k) = 0, one has

$$k = \frac{N \ln(1-p) - \ln(\frac{Q_0 \ln(1-p)(s-c_m)}{c_s})}{\ln(1-p)} . \tag{9}$$

Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for TP at  $k = k^*$ ,  $Q = Q^*$  is

$$\Delta TP(k^*, Q^*) > 0 > \Delta TP(k^* - 1, Q^*),$$
 (10)

where

$$\Delta TP(k^*, Q^*) = \Delta TP(k^* + 1, Q^*) - \Delta TP(k^*, Q^*), \tag{11}$$

### Example 1.

Assuming N=30, p = 0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_s=550$ , and  $Q_o=10000$ .

Then from (9 and 10), k=21.016, because TP(21,Q)=133416 and TP(22,Q)=133414, therefore  $k^*=21$ , with  $Q^*=10947$ , and  $TP(k^*,Q^*)=133416$ .

### Case 2: When the achieve percentage of the retailer's ordering quantity is less than 100%.

In practice, short delivery always happen. Compensation mechanism came into being. In this case, the manufacturer can only set the delivery  $\beta\%$  in the last resort. Then the retailer needs to make compensation  $0.01\beta\delta$  for the manufacturer. When the achieve percentage of the retailer's ordering quantity is  $\beta\%$ , the total profit per manufacturing batch is

$$TP_1(k,Q) = rQ(1-p)^{N-k} + sQ[1-(1-p)^{N-k}] - Qc_m - c_o - kc_s - 0.01(100-\beta)\delta.$$
(12)

Our problem can be formulated as:

Maximize:  $TP_1(k,Q)$ 

Subject to: 
$$Q(1-p)^{N-k} = 0.01 \beta Q_{\alpha}, k \le N.$$
 (13)

From (3),  $TP_1(k,Q)$  can be transformed as

$$TP_{1}x(k) = r0.01\beta Q_{o} + \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}]}{(1 - p)^{N - k}} - \frac{c_{m}0.01\beta Q_{o}}{(1 - p)^{N - k}} - c_{o} - c_{s}k - 0.01(100 - \beta)\delta.$$
(14)

That is, our problem can be formulated as:

Maximize:  $TP_1x(k)$ ,  $k \le N$ . (15)

Since 
$$TP_1x'(k) = \frac{s0.01\beta Q_o[1-(1-p)^{N-k}]\ln(1-p)}{(1-p)^{N-k}} + s0.01\beta Q_o \ln(1-p) - \frac{c_m 0.01\beta Q_o \ln(1-p)}{(1-p)^{N-k}} - c_s.$$
 (16)

$$TP_{1}x''(k) = \frac{s0.01\beta Q_{o}[1 - (1 - p)^{N - k}]\ln^{2}(1 - p)}{(1 - p)^{N - k}} + s0.01\beta Q_{o}\ln^{2}(1 - p) - \frac{c_{m}0.01\beta Q_{o}\ln^{2}(1 - p)}{(1 - p)^{N - k}}.$$
 (17)

Simplify (17), one has

$$TP_1x''(k) = 0.01\beta Q_o \ln^2(1-p)(1-p)^{-(N-k)}(s-c_m) < 0.$$
 (18)

(Note:  $s < c_m$ ) Therefore, which leads to the function  $TP_1x(k)$  being strictly concave.

Set  $TP_1x'(k) = 0$ , one has

$$N \ln(1-p) - \ln(\frac{0.01\beta Q_o \ln(1-p)(s-c_m)}{c_s})$$

$$k = \frac{1}{\ln(1-p)}$$
(19)

Since the number of screening equipment, k, is an integer variable, the necessary condition of optimal solution for  $TP_1 x$  at  $k = k^*$ ,  $Q = Q^*$  is

$$\Delta TP_1(k^*, Q^*) > 0 > \Delta TP_1(k^* - 1, Q^*)$$
 (20)

Example 2.

Assuming N=30, p=0.01, r=25, s=5,  $c_m=10$ ,  $c_o=300$ ,  $c_s=550$ ,  $Q_o=10000$ ,  $\delta=2000$ , and  $\beta=98$ . Then from (19 and 20),

$$k^*=19$$
, with  $Q^*=10946$ , and  $TP(k^*,Q^*)=$128562$ .

### **4 CONSLUSION**

Nowadays, quality requirement is the core competition in business. For the sake of maintaining good quality, the related cost is needed. However, it is important for managers to determine the trade-off between quality and cost. In this study, we derive an optimal order quantity for the raw material and number of screening equipment such that the total profit per manufactured batch is maximized. In analyzing the system, we provide managerial insights to decision makers for planning the optimal number of screening equipment for reducing the quality cost. Illustrative case studies, numerical examples, and sensitivity analysis are presented to demonstrate the proposed model.

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### 四、建議

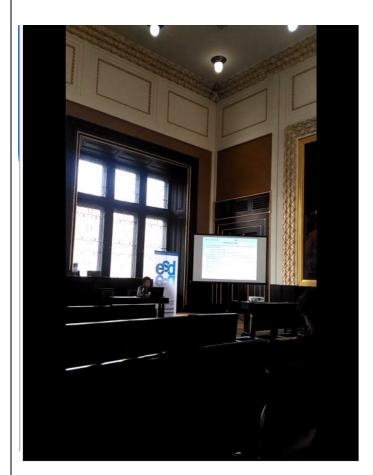
綜上所述,本人得以參與此次研討會,與各國頂尖學者作學術文化交流,也認識許多朋友, 瞭解最近各國研究趨勢,深感獲益良多,更期盼日後仍有機會參與類似國際會議。

### 五、 攜回資料名稱及內容

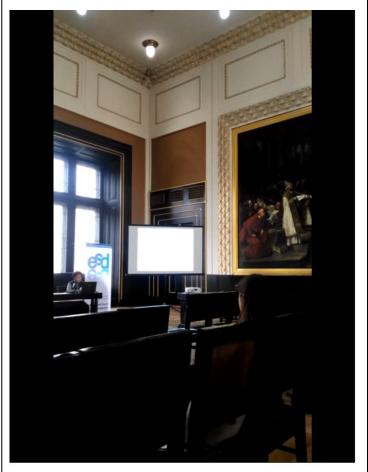
- 1. 研討會論文議程(Proceeding)一冊。
- 2. 研討會論文光碟一片。

## 六、其他

以下為出席國際學術會議剪影:



許炳輝文章發表1



許炳輝文章發表2

### 105年度專題研究計畫成果彙整表

計畫主持人: 滕慧敏 計畫編號: 105-2221-E-263-001-計畫名稱:不完美生產過程檢驗設備策略下之最佳檢驗站數量及製造量 質化 (說明:各成果項目請附佐證資料或細 單位 成果項目 量化 項說明,如期刊名稱、年份、卷期、起 訖頁數、證號...等) 期刊論文 篇 0 研討會論文 0 專書 本 學術性論文 專書論文 0 章 0 技術報告 篇 0 其他 篇 0 申請中 發明專利 0 專利權 已獲得 或 0 新型/設計專利 內 0 商標權 智慧財產權 0 營業秘密 件 及成果 0 積體電路電路布局權 0 著作權 0 品種權 0 其他 0 件數 件 技術移轉 0 千元 收入 期刊論文 本次研討會由International Scientific Conference, Economic and Social Development於2017年4月27日至 4月28日在捷克,布拉格Oldtown Hall 舉行.於4月27日14:30發表,研討會收錄 68 篇論文, 7 場次論文發表。本次研 討會所討論的主題為經濟和社會發展問 題。與會者大多為東歐及少數亞太等知 或 |名學者專家。本人此次發表之題目為 學術性論文 外 : OPTIMAL MANUFACTURING QUANTITY 研討會論文 1 FOR SCREENING EQUIPMENT POLICY WITH IMPERFECT PRODUCTION PROCESS . 大意 為:品質要求已是現代企業的核心競爭 力。為了維持產品的品質,企業必須付 出相對的成本,因此如何取捨品質與成 本的平衡,對管理者相當重要。本研究 建構了數學模式,以求出最佳訂購量及 最佳設備檢驗數量,使總利潤及最佳製 造批量為最佳,研究中並藉由數值範例

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	其他		_	0	篇		
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		積體電路電路布局權			0		
		著作權			0		
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		碩士生			0		
		博士生			0		
參與計畫人力		博士後研究員			0		
		專任助理			0		
	非本國籍	大專生			0		
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		博士後研究員			0		
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## 科技部補助專題研究計畫成果自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值(簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性)、是否適合在學術期刊發表或申請專利、主要發現(簡要敘述成果是否具有政策應用參考價值及具影響公共利益之重大發現)或其他有關價值等,作一綜合評估。

1.	請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估 ■達成目標 □未達成目標(請說明,以100字為限) □實驗失敗 □因故實驗中斷 □其他原因
2.	研究成果在學術期刊發表或申請專利等情形(請於其他欄註明專利及技轉之證號、合約、申請及洽談等詳細資訊)論文:■已發表 □未發表之文稿 □撰寫中 □無專利:□已獲得 □申請中 ■無技轉:□已技轉 □洽談中 ■無其他:(以200字為限)
3.	請依學術成就、技術創新、社會影響等方面,評估研究成果之學術或應用價值 (簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性,以500字為限) 品質需求儼然成為現今企業的核心競爭力,因此如何維持好的品質所投入的相關成本是必要的,而權衡品質與成本,對管理者是非常重要的,本研究藉由數學模式的建立,推導最佳的品質管理檢驗設備數量,期使產品的良率及投入成本平衡.模式分為兩種型態分別討論: 1. 零售商訂購量100%的達成 2. 零售商訂購量無法完全供應
4.	主要發現本研究具有政策應用參考價值:■否 □是,建議提供機關(勾選「是」者,請列舉建議可提供施政參考之業務主管機關)本研究具影響公共利益之重大發現:□否 □是 說明:(以150字為限)